Rejection and valuations

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Timothy Smiley's wonderful paper 'Rejection' (*Analysis*, 1996) is still perhaps not as well known or well understood as it should be. This quick note first gives a quick presentation of themes from that paper, though done in my own way, and then considers a putative line of objection to one key claim in Smiley's paper.

1 The bilateralist framework

Imagine a speech community for whom any sentence is explicitly structured into a propositional content clause and a force indicator. We could imagine that our community has quite a rich repertoire of force-indicators, including markers to indicate sentences whose primary serious uses have imperatival, interrogative, and optative force. But we're going to be concentrating on just two of their force-markers, forming a complementary pair – one to mark sentences whose primary serious use is as an *assertion* of a given propositional content, one to mark a corresponding *rejection*.

If we want to mock up in English a simulacrum for such bipartite forms of utterance, we can't do much better than Smiley's suggestion – a shoulder-shrugging content-expression, followed by 'Yes' or 'No'. 'Today is Saturday? Yes!'. 'It is warm? No!'

I will use '$+$' and '$-$' to represent our community's two force-markers, and then '$+P$' and '$-P$' will represent their respective sentences whose primary uses are for asserting and rejecting the content P. Smiley unhelpfully, and potentially very misleadingly, leaves the force-marker for assertion tacit, and just stars rejected contents. We could use '$\vdash P$' and '$\dashv P$', but that has its own drawbacks. For we might want to consider reasoners who use such sentences in 'off-line' mode, in suppositional reasoning, so the assertorically marked P is not actually asserted – and using a conventional assertion sign might look distinctly odd in such cases. The less committal use of plus and minus signs in formal representations, following e.g. Rumfitt, is therefore an improvement, though the danger remains that '$-P$' now looks rather too much just like a kind of negation of P.

Now, what *makes* our imagined community's pair of force-markers form sentences whose primary use is for assertion and rejection respectively? How they are used, of course: how utterances involving them are linked into the intentional states of speakers and their audiences. Perhaps, very roughly, the default serious use of '$+P$' conventionally indicates a high level of credence in P (or an intention to get one's audience to give high credence to the content P, or some such). Correspondingly, the default serious use of '$-P$' indicates a low level of credence in P (or an intention to to get one's audience to give low credence to P, or whatever). The details of how we spell that out don't really matter, so long as it has two features. First, the account should make it clear that the act of rejection can be explicated without reference to the operation of negation applied to contents. And second, it should also make it clear that the two utterences

'$+P$' and '$-P$' are conventionally tied in their primary uses to propositional attitudes which cannot be held together as attitudes to the same content. A joint utterance of '$+P$' and '$-P$' (seriously, at the same time, in the same context) would therefore be a serious conversational mishap – the utterance of the one must undermine the standard purposes of the utterance of the other. The utterances cancel each other out. Our community don't allow serious joint utterances as a satisfactory conversational resting place.

Imagine next that our speech community has a certain practice of reason-giving in defence of assertions and rejections (what makes a sequence of assertions and rejections into a reasoning practice is again a matter of use, of how it engages with their intentional states). Commenting from the sidelines, we note that they treat certain classes of reasons as conclusive (in the sense of not to be undermined by further reasons), and that they freely chain such reasons. So we can here describe them as operating with a consequence relation which admits of weakening and a cut rule – and we can throw in reflexivity to describe the null case (if they already accept/reject P, they don't wait upon a further reason for accepting/rejecting P).

But in addition to these familiar structural rules for a consequence relation – reflexivity, weaking, cut – there'll be another rule structural needed to describe our community's practice, given that their utterances come in two flavours, overtly tagged as assertions or rejections. For suppose someone reasons from given premisses to both $+P$ and $-P$. Then, by their own lights, they have got themselves into a tangle and must recognize that something has to go. If we use lower case Greek letters to indicate whole signed sentences and $^*\alpha$ for the result of changing α's force-marker to the opposite one: then the community will conform to the rule that counts a proof from some signed premisses including α to β plus another proof from the same premisses including α to $^*\beta$ as together yielding a proof from the remaining premisses to $^*\alpha$. In a standard representation, the rule is

$$\frac{\Gamma \qquad \begin{array}{c}[\alpha]\\ \vdots\\ \beta\end{array} \qquad \begin{array}{c}[\alpha]\\ \vdots\\ ^*\beta\end{array}}{^*\alpha}$$

Call that Smileian Reductio – but note again that this rule is a *structural* rule, unlike more familiar kinds of reductio. Note too we might suppose that our community, wondering whether to say $+P$ or $-P$, engage in such reasoning in suppositional mode. Against the background Γ, they try accepting P for size, and see what that commits them to, and come to reject P.

As Smiley then points out, this form of Reductio immediately delivers, given the other structural rules, the Reversal Principle that converts a proof from Γ, α to β to a proof from $\Gamma, {}^*\beta$ to $^*\alpha$.

So far, there is no explicit presumption that our community use propositional logic (their reasoning so far could be all be syllogistic, for example). But our community can now introduce propositional connectives into that given 'bilateral' inferential framework, and in a particularly elegant way. (It is not, however, of the essence that you fully buy this just-so story, with an inferential practice supposed to be up and running before propositional reasoning gets off the ground. What we are dramatizing is the thought that general structural features of reasoning are one thing, rules governing particular logical operators are something else.) The introduction rules are

1. from $+P$, $+Q$, infer $+(P \wedge Q)$,

2. from $-P$, $-Q$, infer $-(P \vee Q)$,

3. from $-P$ infer $+\neg P$.

Note the difference between (i) $-P$, (ii) $+P$, and (iii) $+\neg P$. (i) and (ii) express different attitudes to the same content; (ii) and (iii) express the same attitudes to different contents, and by the same token (i) and (iii) express different attitudes to different contents and so are not to be confused.

And now here are the evidently harmonious elimination rules that enable us to backtrack from a complex formula to extract what the introduction rule governing its main connective required us to put in:

1. from $+(P \wedge Q)$, infer $+P$: from $+(P \wedge Q)$, infer $+Q$:

2. from $-(P \vee Q)$, infer $-P$: from $-(P \vee Q)$, infer $-Q$:

3. from $+\neg P$ infer $-P$.

This system of rules is as simple and remarkably elegant as can be (note in particular the absence of a primitive absurdity sign). Two initial results about it.

First, negation elimination rule tells us that from $+\neg\neg P$ we can infer $-\neg P$; but applying the reversal principle to negation introduction tells us that from $-\neg P$ we can infer $+P$; so cut gives us double negation elimination for positive formulae. And in fact, in the presence of the structural rules (including Smilean reductio and hence reversal), the rules enable us to warrant *all* the usual classical deductions involve inferences from positively signed formulae to a positive conclusion. Moreover, we get that classical upshot using entirely harmonious negation rules – unlike, of course, the situation with standard propositional logics.

Second, we'll say, in the usual way, that a two-valued truth-valuation of propositional contents is Boolean if it respects the classical truth-tables for the connectives (though it needn't respect other, community-accepted, reasonings). And we'll say that an inference involving signed premisses and conclusion is *classically$_b$ valid* – the subscript 'b' is for 'bilaterally' – if every Boolean assignment of truth-values to propositional contents, which makes all plus-signed premisses true and minus-signed premisses false, makes the conclusion true if plus-signed and conclusion false if minus-signed.

We can define validity more snappily, if we help ourselves to a bit of derivative terminology. We'll say that a positively signed sentence is correct if its content is valued as true, and incorrect if the content is valued false; and dually for negatively signed sentences. Then we can equivalently say that an inference is classically$_b$ valid if any Boolean valuation of contents which makes the premisses correct makes the conclusion correct. (But do note here the important point that valuations are still over *contents*, as of course they should be.)

The key second result is that our simple inference rules are sound and complete for classically$_b$ valid signed inferences.

2 Bilateralism and the Carnap point

Let's say that a valuation of propositional contents is hyper-Boolean if it is either Boolean or it makes every content simultaneously true. And let's say a signed inference is hyper-classically$_b$ valid if any hyper-Boolean valuation of contents which makes the premisses correct makes the conclusion correct. Then obviously the elimination rule (E3), from

$+\neg P$ infer $-P$, is *not* sound for hyper-classically$_b$ validity. For take the valuation which makes every content simultaneously true. Then $+\neg P$ is correct, $-P$ is incorrect, and the rule takes us from a correct premiss to an incorrect conclusion. So our signed inferential rules are true to classical$_b$, but not hyperclassical$_b$, validity.

Here there is a lovely contrast with standard classical systems (meaning familiar propositional logics dealing, in effect, with just positively signed sentences). Say a unilateral inference is is hyper-classically valid if any hyper-Boolean valuation which makes the premisses correct makes the conclusion correct. Then classical systems are not only sound and complete for classical validity but for hypervalidity too, as Carnap pointed out. Why so? Because if an inference is classically valid, i.e. there is no valuation which makes the premisses true and conclusion false, then it is hypervalid too (as the additional valuation we need to consider never makes a conclusion false). And if an inference is not classically valid, then the valuation which makes the premisses true and conclusion false also counts as a hyper-valuation which makes the premisses true and conclusion false, so the inference is not hyperclassically valid. The two notions of validity are co-extensional in a unilateral framework.

Let's define *weak inferentialism* about a class of logical operators to be the doctrine that fixing the inference rules governing those operators fixes all the logically salient properties of sentences containing the operators (e.g. which propositions are consistent with which, which are contraries, etc. etc.). And define *strong inferentialism* about a class of logical operators to be the doctrine that fixing the inference rules fixes the meaning of the operators. Smiley's interest in his paper is in weak inferentialism. And what we've just seen is that, in a standard unilateral framework, weak inferentialism is false for the propositional connectives. Accepting the standard rules doesn't rule out a deviant semantics which in fact allows a valuation where P and $\neg P$ both come out true, so making P and $\neg P$ semantically consistent.

By contrast, in the bilateral framework, our rules do rule out e.g. a valuation which makes $+P$ and $+\neg P$ simultaneously true. So the Carnap problem – or at least his problem for weak inferentialism that arises from considering hypervaluations – doesn't arise. And Smiley in fact gives a more general 'categoricity' proof: "all the traditional semantically defined logical notions have purely deductive equivalents", and so get correctly fixed by the bilateral inference rules.

Evidently, *strong* inferentialism implies *weak* inferentialism, so if the latter is false, so is the former. That's why the Carnap point impacts on strong inferentialism. And if we can protect weak inferentialism by going bilateral, to that extent we protect strong inferentialism. Of course weak does not entail strong, and we might indeed have various doubts about the strong version. But this isn't the place to raise them. All we are claiming is that, in the bilateral framework, the Carnap point against inferentialisms can be resisted.

But what's all that to us, you might ask? After all, we don't speak and infer bilaterally, do we? But perhaps arguably we do: perhaps some of our negative talk, some of our 'not's and many of 'no's and all our 'no way!'s, are actually operating as rejective force-markers and not as propositional-content modifiers. Or at least, that has to be Smiley's implicit claim. And that's what will need to be investigated if the philosophical impact of his technical observations about bilateral inference rules is to be assessed. To be sure, Smiley himself doesn't push the investigation very far in his paper, other than remove a Fregean road-block: but others (Price, Rumfitt, Humberstone) have done more.

3 Can Smiley be Carnapped?

Suppose we present the definition of classical$_b$ validity in a truncated way like this: an inference is classically$_b$ valid if every valuation which makes the premisses correct makes the conclusion correct. Then that might encourage you[1] to wonder: if we are allowed to consider a truth-valuation which make all propositional contents true together, in order to make trouble for weak inferentialism in that setting, why shouldn't we be allowed to consider a correctness-valuation which makes all signed sentences correct together to make trouble for Smiley's defence? It might be asked: how do Smiley's rules rule out a rogue everything-is-correct valuation? So it would seem that Carnap's problem can just be shifted to be replayed in terms of correctness-valuations.

But that's a mistake. It is propositional *contents* that are the primary locus of evaluation, and it is in terms of such an evaluation that validity is being basically defined: and Smiley's rules ensure that negation – which is, remember, an operation on *contents* – behaves as we want. True, a positively signed sentence can then *derivatively* be said to be correct if it has a true propositional content, and a negatively signed sentence is derivatively correct if it has a false propositional content, and so on. So we can a give a derivative account of classical$_b$ validity in terms of correctness-preservation. But, so defined, there just can't be a correctness-valuation which makes all signed sentences true together – for by our starting hypothesis we cannot simultaneously correctly assert and correctly reject the same content. So the alleged problem doesn't arise.

'Hold on,' you might respond, 'we were allowed to put the meaning of negation on hold and make play with a rogue truth-valuation which made all contents true together in the original Carnapian argument: why can't we now put on hold the exclusivity of assertion and rejection in running a Carnapian argument against Smiley's claim that his bilateral rules succeed where the unilateral classical rules fail?' But this response would just compound the mistake. In the original context, we are wondering whether the content of the negation sign was indeed fixed by the rules of inference, so of course we put its intended meaning on hold while ask just which valuations are consistent with the inferential practice. But now, in considering Smiley's discussion, we are wondering whether the content of the negation sign is fixed by the bilateral inferential practice *when added to a given background of the use of force-markers to construct sentences whose default use is for assertion and rejection.* The meaning of those markers isn't up for revision, it is part of the assumed background. And so it is a given that we cannot simultaneously correctly use $+P$ and $-P$.

Of course, one may be unhappy with the idea that there is a speech act of rejection distinguishable from the act of asserting a negation. You can read the formalism so that the minus-sign in $-P$ is just a kind of non-embeddable negation. But to do so would not be an *objection* to Smiley, but a flat unargued rejection of his starting point.

[1]The 'you' might include Julien Murzi and Ole Hjortland in a talk at the Second Cambridge Graduate Conference on the Philosophy of Logic and Mathematics, Jan. 2009.