Metaphysics of Modality

Lecture 3
Today’s Lecture

• Abstract Realism
  – There are possible worlds
  – Possible worlds are abstract objects of some sort or other

(1) Possible worlds as sets of sentences

(2) Possible worlds as states of affairs
Abstract Realism – Worlds as Sets of Sentences

Possible Worlds

\[ x \text{ is a possible world} \iff x \text{ is a maximal consistent set of sentences.} \]

\[ p \text{ is true at a possible world } w \iff p \in w \]
Abstract Realism – Worlds as Sets of Sentences

Possible Worlds

$x$ is a possible world $\iff x$ is a maximal consistent set of sentences.

- Not any arbitrary set of sentences is a possible world:
  - $\{Pa, \neg Pa\}$

- To be a possible world a set of sentences must be consistent.
Abstract Realism – Worlds as Sets of Sentences

Possible Worlds

\( x \) is a possible world \( \iff x \) is a maximal consistent set of sentences.

- Not any arbitrary consistent set of sentences is a possible world:
  - \( \{Pa\} \)

- To be a possible world a set of consistent sentences must be \textit{maximal}.

- \( S \) is maximal iff
  - for every atomic sentence \( p \), \( S \) contains either \( p \) or \( \neg p \).
  - \( S \) contains all the sentences entailed by these atomic sentences.

- Suppose our only two atomic sentences are \( Pa \) and \( Fb \):
  - A maximal set would be e.g. \( \{Pa, \neg Fb, Pa \& \neg Fb, \exists x Px, \ldots \} \)
Abstract Realism – Worlds as Sets of Sentences

1. Problem of Reduction

• Consistency is typically spelled out in modal terms:

• A set of sentences $S$ is consistent iff it is possible that all the sentences in $S$ are true together

• Can we spell out consistency without appealing to modality?

• Perhaps we can characterise consistency syntactically:

• A set of sentences $S$ is consistent iff there is no derivation of a contradiction from the sentences it contains in FOL.

• $\{Pa, \neg Pa\}$ is inconsistent because we can derive $Pa$ and $\neg Pa$
• By spelling out consistency syntactically, we can make sure that every sentence that is logically impossible is false at every world.

• But some sentences should be false at every world without being logically impossible.
  – $a$ is (entirely and always) red & $a$ is (entirely and always) blue
  – $Ra$ & $Ba$
  – We cannot derive a contradiction from \{Ra, Ba\} in FOL

• Just add the following as a non-logical axiom to FOL:
  \[(A1) \forall x \neg(Rx \& Bx)\]

• A set of sentences $S$ is consistent iff there is no derivation of a contradiction from the sentences it contains in FOL + (A1).
For this strategy to work we will have to add further axioms to ensure that no non-logical impossibility holds at any world.

We can’t simply say:
(M) If $p$ is a necessary truths then $p$ is an axiom

But is there really an alternative?

Our language has to be rich enough to talk about both micro-physical objects (electrons) and macro-physical objects (elephants).

Consider a maximal set that contains
- micro-physical sentences to the effect that there is an elephant at place $p$ at time $t$.
- macro-physical sentences to the effect that there is no elephant at place $p$ at time $t$. 
1. Problem of Reduction

• To make sure that such a set is inconsistent we would have to add an axiom of the following form:

\[(A2) \text{ [Micro]} \rightarrow \text{ [Macro]}\]

Roughly:
If something fulfils a certain micro-physical description D then it is an elephant

• Now the problem is that we will need very many such bridge laws

• There’s no guarantee that finitely many will do

• Even if finitely many bridge laws suffice, no one will ever be able to state all of them

• So the only theory that has actually been stated and that is reductive remains Lewis’s.
Abstract Realism – Worlds as Sets of Sentences

2. Problem of Expressive Power

• Arguably there are as many spacetime points as there are real numbers

• For each of the points it is possible that only that point is occupied by matter while all other points are vacant

• To represent all these possibilities we need a name for each of the points

• Lagadonian Language:
  – Each individual serves as a name for itself
  – Each property serves as a predicate

• The Lagadonian “sentence” saying that Anna is a footballer:
  <Anna, the property of being a footballer>
Abstract Realism – Worlds as Sets of Sentences

2. Problem of Expressive Power

- What about possibilities involving individuals that don’t actually exist?
- There could have been a person who doesn’t actually exist and who is a footballer.
- Even in a Lagadonian language we don’t have a name for that merely possible individual
- We need to make do with a descriptive sentence: “there is an individual which is distinct from, Tom, Dick, Harry ... and who is a footballer”
- Worlds with non-actual individuals are constructed with the help of descriptions that name only actual individuals and qualitative properties
2. Problem of Expressive Power

• There ought to be a world $w_1$ according to which there are two non-actual individuals who are just like Batman and Robin are according to the story:

• “there is an individual which is distinct from, Tom, Dick, Harry … and who wears a bat-mask, … and who is accompanied by another guy who wears a yellow cape…”

• Is there another world $w_2$ which is just like $w_1$ except that the guy who plays the Batman-role in $w_1$ plays the Robin-role in $w_2$ and vice versa?

• If you are a haecceitist: yes!

• “there is an individual which is distinct from, Tom, Dick, Harry … and who wears a bat-mask, … and who is accompanied by another guy who wears a yellow cape…”
Abstract Realism – Worlds as States of Affairs

Possible Worlds

\( x \) is a possible world \( \iff x \) is a maximal consistent state of affairs.

• Wittgenstein’s being a philosopher.
• Rooney’s being a footballer.
  – These are states of affairs that obtain.

• Wittgenstein’s being a footballer.
• Rooney’s being a philosopher.
  – These are states of affairs that don’t obtain.
  – But they exist just as much as those which do obtain.
Abstract Realism – Worlds as States of Affairs

Possible Worlds

$x$ is a possible world $\iff x$ is a maximal consistent state of affairs.

- Not any arbitrary state of affairs is a possible world:
  - Wittgenstein’s being a philosopher and not being a philosopher.
- To be a possible world a state of affairs must be consistent.
Possible Worlds

\( x \) is a possible world \( \iff \) \( x \) is a maximal consistent state of affairs.

- Not any arbitrary consistent state of affairs is a possible world:
  - Wittgenstein’s being a philosopher.
- To be a possible world a state of affairs must be \textit{maximal}.
Abstract Realism – Worlds as States of Affairs

Possible Worlds

• Consistency and maximality are defined as follows:

• $S$ is consistent $\iff$ It is possible that $S$ obtains

• $S$ is maximal $\iff$ For every state of affairs $S^*$, $S$ either includes $S^*$ or precludes $S^*$

• $S$ includes $S^*$ $\iff$ It is impossible that $S$ obtains and $S^*$ doesn’t.

• $S$ precludes $S^*$ $\iff$ It is impossible that $S$ and $S^*$ both obtain.

• $p$ is true at a maximal consistent state of affairs $S$ $\iff$
  Necessarily (if $S$ obtains then $p$)
Abstract Realism – Worlds as States of Affairs

1. Problem of Reduction

• Consistency and maximality are defined as follows:

• $S$ is consistent $\iff$ It is possible that $S$ obtains

• $S$ is maximal $\iff$ For every state of affairs $S^*$, $S$ either includes $S^*$ or precludes $S^*$

• $S$ includes $S^*$ $\iff$ It is impossible that $S$ obtains and $S^*$ doesn’t.

• $S$ precludes $S^*$ $\iff$ It is impossible that $S$ and $S^*$ both obtain.

• $p$ is true at a maximal consistent state of affairs $S$ $\iff$ Necessarily (if $S$ obtains then $p$)
2. Problem of Expressive Power

- For every actual and non-actual individual there is a corresponding individual essence.

- The individual essence of $x$ is a property $F$ such that
  (i) $x$ exemplifies $F$ at every world at which it exists
  (ii) no individual other than $x$ exemplifies $F$ at any world.

- The individual essence of a contingent object is not exemplified at every world; but (like every property) it exists at every world.

- At the actual world there exists an (unexemplified) individual essence for each non-actual individual.

- Let $E_1$ and $E_2$ be two such individual essences.
2. Problem of Expressive Power

- There is a maximal consistent state of affairs $S$ such that
  
  (i) Necessarily (if $S$ obtains then there is an object which exemplifies $E_1$ and also exemplifies wearing a bat mask, … )

  (ii) Necessarily (if $S$ obtains then there is an object which exemplifies $E_2$ and also exemplifies wearing a yellow cape, … )

- There is a maximal consistent state of affairs $S$ such that
  
  (i) Necessarily (if $S$ obtains then there is an object which exemplifies $E_2$ and also exemplifies wearing a bat mask, … )

  (ii) Necessarily (if $S$ obtains then there is an object which exemplifies $E_1$ and also exemplifies wearing a yellow cape, … )
Abstract Realism

3. The Objection from Magic

• S represents that \( p \iff \text{Necessarily (if } S \text{ obtains then } p) \)
• S represents that there is a blue elephant\( \iff \)
  Necessarily, if S obtains then there is a blue elephant
• Consider the following states of affairs:
  – The (obtaining) state of affairs \( S_1 \) of there being a grey elephant
  – The (not-obtaining) state of affairs \( S_2 \) there being a blue elephant
• @ stands in a relation to \( S_1 \) in which it doesn’t stand to \( S_2 \)
• Lewis calls this the selection relation
• There is something problematic about this selection relation
Abstract Realism

3. The Objection from Magic

Selection Relation

Internal

External
Abstract Realism

3. The Objection from Magic

$xRy$ is internal $\iff$ $R$ holds in virtue of the intrinsic nature of the relata $x$ and $y$

- If an internal relation holds between two objects $x$ and $y$, then it also holds between any perfect duplicate of $x$ and any perfect duplicate of $y$
- *Example:* $x$ is taller than $y$

$xRy$ is external $\iff$ $R$ doesn’t hold in virtue of the intrinsic nature of the relata $x$ and $y$

- An external relation can hold between two objects $x$ and $y$ while not holding between a perfect duplicate of $x$ and a perfect duplicate of $y$
- *Example:* $x$ is 1m apart from $y$
Abstract Realism

3. The Objection from Magic

Selection Relation

Internal

External

Magical connection between @ and S1
Abstract Realism

3. The Objection from Magic – External Horn

• We’re supposing that the selection relation between @ and S1 is external

• Necessarily (there are grey elephants at @ ↔ @ selects S1)

• An intrinsic aspect of one thing is necessarily connected with that thing standing in an external relation to some other thing

• According to Lewis such necessary connections are magical

• Necessarily (the table is wooden ↔ the table is 1m apart from the chair)

• Note: it’s not the relation of selection itself which holds necessarily

• It’s the relation: If there are grey elephants at x then x selects y
3. The Objection from Magic

Our grasp of the selection relation is magical

Magical connection between @ and S1
Abstract Realism

3. The Objection from Magic – Internal Horn

- We’re supposing that the selection relation between @ and $S_1$ is *internal*

- $S_1$ must have some intrinsic property in virtue of which it rather than $S_2$ is selected by @ if there is a grey elephant

- What’s that intrinsic property?

- A representational property. In particular: The property $P$ of representing that there is a grey elephant

- Now the abstract realist owes us an account of what such representational properties are
3. The Objection from Magic

Selection Relation

Internal

$S1$ has intrinsic representational property $P$

Acquaintance with $P$

External

Magical connection between @ and $S1$
Abstract Realism

3. The Objection from Magic – Internal Horn

• $S_1$ must have some intrinsic property in virtue of which it rather than $S_2$ is selected by @ if there is a grey elephant

• What’s that intrinsic property?

• A representational property. In particular:
  The property $P$ of representing that there is a grey elephant

• Now the abstract realist owes us an account of what such representational properties are

• Can we be acquainted with $P$?

• No, for by definition $S_1$ is abstract
Abstract Realism

3. The Objection from Magic

Selection Relation

Internal

S1 has intrinsic representational property $P$

Acquaintance with $P$

External

Magical connection between @ and S1

Analysis of $P$
3. The Objection from Magic – Internal Horn

- We cannot be acquainted with $P$
- $S1$ must have some intrinsic property in virtue of which it rather than $S2$ is selected by @ if there is a grey elephant
- What’s that intrinsic property?
- A representational property. In particular:
  The property $P$ of representing that there is a grey elephant
- Now the abstract realist owes us an account of what such representational properties are
- Can we provide an analysis of $P$? Lewis finds the following not informative enough:
  - $P$ is the property of representing that a there is a grey elephant $\leftrightarrow$
    Necessarily ($S$ has $P$ & $S$ is selected $\rightarrow$ there is a grey elephant)
Abstract Realism

3. The Objection from Magic

Selection Relation

Internal

S1 has intrinsic representational property P

Acquaintance with P

Our grasp of the selection relation is magical

Analysis of P

External

Magical connection between @ and S1
Abstract Realism

3. The Objection from Magic ... overgenerates

• Consider the relation of set-membership: \( x \in y \)

• It’s not an internal, but an external relation:
  – Note that \{John\} is a duplicate of itself
  – Let John* be a distinct duplicate of John
  – John \( \in \) {John} but John* \( \notin \) {John}

• Consider the set \{x: x is wooden\}

• Necessarily (if x is wooden then x \( \in \) \{x: x is wooden\})

• An intrinsic aspect of one thing is necessarily connected with that thing standing in an external relation to some other thing

• Set-membership gives rise to magical connections just as much the selection relation (if assumed to be external) does!