

Logical Form

Formal and material validity

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Logical Form

- ▶ Logical form is a crucial notion in philosophical logic.
- ▶ It is often discussed, but rarely the focus of discussion.
- ▶ We'll impose some structure on the debate, and reflect on logic's purpose, status and history.
- ▶ Our plan:
 1. Formal and material validity
 2. Davidson on logical form
 3. Logical form and grammatical form
 4. The metaphysics of forms

Talk outline

Validity

Formal and material validity

Natural and formal language

Conclusion

Necessary truth-preservation

- ▶ How might we understand validity in English?
 - N An argument is valid iff, necessarily, if the premises are true, then the conclusion is true.
- ▶ Consider:
 - ▶ The ball is round and the ball is green; so the ball is green
 - ▶ Socrates is human, all humans are mortal; so Socrates is mortal
 - ▶ Mahershala Ali won an Academy Award; so Mahershala Ali won an Oscar

Model-theoretic validity

- ▶ How might we understand validity in FOL?
 - (M) An argument is valid iff every model of the premises is a model of the conclusion.
- ▶ Reminder:
 - ▶ A model of a sentence is an interpretation in which that sentence is true.
 - ▶ A model of a set of sentences is an interpretation in which all of the sentences in that set are true.
 - ▶ An interpretation is an ordered pair, $\langle D, V \rangle$ of a domain, D , and a valuation function, V .
 - ▶ The domain is a non-empty set of objects.
 - ▶ The valuation function assigns objects from the domain to names, sets of objects to one-place predicates, sets of ordered pairs to the two-place predicates and, generally, sets of ordered n-tuples to n-place predicates.

Formalisation

- ▶ The ball is round and the ball is green; so the ball is green
 - ▶ $Rb \wedge Gb \therefore Gb$ – valid
- ▶ Socrates is human, all humans are mortal; so Socrates is mortal
 - ▶ $Hs, \forall x(Hx \rightarrow Mx) \therefore Ms$ – valid
- ▶ Mahershala Ali won an Academy Award; so Mahershala Ali won an Oscar
 - ▶ $Aa \therefore Oa$ – invalid

Options

- ▶ What to do?
 1. Change logic: to the extent that this argument cannot be captured, our logic is deficient.
 2. Change formalisation: FOL is correct and we must instead change the formalisation.
 3. Ignore: this argument is not the concern of logic. We need a story to explain this.

Option 1: change logic

- ▶ Consider the following arguments:
 1. It is necessary that $2+2=4$; so it is possible that $2+2=4$.
 2. The Brontë sisters supported one another, the Brontë sisters were Anne, Charlotte and Emily; so Anne, Charlotte and Emily supported one another.
 3. Tim is a lecturer, Owen is a lecturer; so there is something that Tim and Owen both are.
- ▶ These are all valid by (N) but not by (M).
- ▶ They can be made valid by natural extensions of (M).

Option 1: change logic

- ▶ Mahershala Ali won an Academy Award; so Mahershala Ali won an Oscar
 - ▶ $Aa \therefore Oa$ – invalid
- ▶ We could take ‘x won an Academy Award’ and ‘x won an Oscar’ as logical constants.
- ▶ Then the argument would be valid by (N) and its formalisation by (M).

Option 2: change formalisation

- ▶ Or we could keep the logic, and change the translation.
Consider:
 - ▶ The winner of the best actress Golden Globe is Olivia Coleman;
so there is exactly one winner of the best actress Golden Globe.
- ▶ This is valid by (N).
- ▶ To give it an (M)-valid formalisation, we could offer:
 - ▶ $\exists x \forall y ((Gy \leftrightarrow y = x) \wedge x = c) \therefore \exists x \forall y (Gy \leftrightarrow y = x)$

Option 2: change formalisation

- ▶ Mahershala Ali won an Academy Award; so Mahershala Ali won an Oscar
 - ▶ $Aa \therefore Oa$ – invalid
- ▶ We could choose to formalise 'x won an Academy Award' and 'x won an Oscar' as the same predicate.
 - ▶ $Aa \therefore Aa$

Option 3: ignore

- ▶ Suggestion:

Some arguments, although valid in the sense of (N), are not the sorts of arguments that logicians should be out to capture, so it needn't bother us if they are not valid by (M).

- ▶ Some arguments, as well as being valid by (N), are also *formally* valid.
- ▶ Arguments valid by (N) but not formally valid are *materially* valid.

Option 3: ignore

- ▶ Mahershala Ali won an Academy Award; so Mahershala Ali won an Oscar
- ▶ Consider the form in virtue of which this may be valid:
 - ▶ x won an Academy Award; so x won an Oscar
- ▶ Surely Academy Awards and Oscars are not the domain of logic.

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Some history

- ▶ The distinction between *formal* and *material* validity first appears in medieval logic.
- ▶ It is thought that the terms were first used by William of Ockham in the early 14th century in his *Summa Logicae*.
- ▶ The clearest medieval statement of the distinction is found in Jean Buridan's 14th century work *Treatise on consequences*:

Formal consequence means that the consequence holds for all terms, retaining the form common to all.
- ▶ He gives 'A man runs; so an animal runs' as an example of material consequence.
- ▶ Not formally valid: 'A horse walks; so wood walks'.
- ▶ Crucially, this distinction is in *natural language*.

Purely valid forms

- ▶ How can we cash out this notion of formal validity?
- ▶ First attempt: an argument A is formally valid iff it has a purely valid form.
- ▶ An argument-form is purely valid just if every argument of that form is valid.
- ▶ The ball is round and the ball is red; so the ball is red
 - ▶ A and $B \therefore B$ – valid
- ▶ All arguments of that form are valid.

Logical forms

- ▶ Problem: 'x won an Academy Award; so x won an Oscar' is a purely valid form.
- ▶ Second attempt: an argument A is formally valid iff it has a purely valid *logical* form.
- ▶ Although 'x won an Academy Award; so x won an Oscar' is purely valid, it is not a *logical* form.
- ▶ 'x won an Academy Award' and 'x won an Oscar' are not *logical*.

Logical vocabulary

- ▶ We now need a distinction between logical and nonlogical vocabulary *in natural language*.
- ▶ Even in *formal* languages, this is controversial.
- ▶ E.g. identity, higher-order quantifiers, modal operators
- ▶ Applied to natural language, it's even less clear what this could mean.
- ▶ If we settled the question for formal languages, we could appeal to formalisation.

Further Problems: Strawson

- ▶ Strawson presents further problems for this account in his *Introduction to Logical Theory* (1952).
- ▶ He (1952: 42) argues that

“It is and it is not” may be held to exemplify the verbal pattern “*P* and not-*P*” and yet may be used to give a perfectly consistent answer to a question (e.g., “*Is it raining?*”)
- ▶ ‘It is and it is not’ would normally be formalised as ‘ $P \wedge \neg P$ ’ which is, classically, logically false.
- ▶ In English this is not exceptionless.

Further Problems: Cargile

- ▶ Cargile takes the Strawsonian line further in 'Logical Form' (2010).
- ▶ Even and-elimination, Cargile believes, fails to be exceptionless in English
 - I'm happy and I'm sad; so I'm happy
- ▶ Cargile:

what the practicality of English is incompatible with is mathematically precise pairing of syntactic structure with logical functions (2010: 92)

Further problems: Cargile

- ▶ Later:

what the practicality of English is incompatible with is the view that validity is entirely due to a logical form inherent in the sentences of the argument (113)

- ▶ In English, the rules

were “made to be broken” by poets, or philosophers, or simple people needing charitable hearing (92)

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A different perspective

- ▶ Alex Oliver in 'The Matter of Form: Logic's Beginnings' (2010):

Why, then, do we not capture these kinds of inferential behaviour when we put arguments into logical form? It is wrong to look for the answer on the side of natural language, as if these inferences lacked some special kind of validity. No: the answer is on the side of the models, i.e. the formalized languages. We capture those inferences that our models can capture. (343)

- ▶ So the correct answer may lie on the formal, rather than natural, side of the linguistic divide.

Mathematical mileage

- ▶ *Why* have our formal logical practices developed this way?
- ▶ Is it just a matter of the accidental history?
- ▶ Timothy Smiley, in 'A tale of two tortoises' (1995) thinks not:
'Older than' or 'married to' don't get in for the simple reason that there isn't enough mathematical mileage in them, while sets and recursive functions have hived off for the opposite reason. (735)

Further work

- ▶ We now need an account of mathematical interest.
- ▶ But consider Arthur Prior's 'What is logic?' (1976):
there is some kind of gradation in these things – that there is somehow more point, for example, in talking about a logic of time and tenses than there is in talking about a logic of organic life (129)
- ▶ To determine whether a logic is worth developing, we must
try it out and see what happens. You can't settle the question a priori (129)

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- ▶ We started with an apparently valid argument that FOL misses.
- ▶ Three possibilities: change logic, change formalisation, or ignore it
- ▶ The most popular way of ignoring such arguments appeals to the material/ formal distinction.
- ▶ This distinction cannot be spelt out without considerable difficulty.
- ▶ An intriguing suggestion: try to capture the argument, see what happens and, if the resulting logic proves useful and interesting, change our models accordingly.
- ▶ This answer is on the side of formal language.