Logical Form
The metaphysics of forms

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Some theories propose logical forms that deviate greatly from surface grammatical form.

This has led to the slogan that ‘grammatical form misleads as to logical form’.

The paradigmatic case is quantifier phrases and proper names.

This case is a myth: they are not of the same grammatical form.

And the mistake is *logical* rather than *grammatical*.

Further, the existence of logically perfect languages is doubtful: even TFL fails to meet the standard
Talk outline

Form bearers

What are forms?

The Schematic Fallacy

Realism vs instrumentalism

Conclusion
Two questions

- We’ve been discussing logical forms.
- *What* have we been talking about?
- Two questions:
  1. What sorts of entities *have* logical forms?
  2. What sorts of entities *are* logical forms?
- Let’s take them in turn.
Form bearers

- We’ve generally used *sentences* as form bearers.
- Some defend *propositions*.

**Sentence** A string of symbols constructed according to a grammar. They can be uninterpreted or interpreted.

**Proposition** An ordered entity consisting of worldly items, not tied to any particular language and against which sentences may be true or false.
Sentences

- Logical form is intimately related to logical consequence/validity.
- Two features in determining forms are the *logical constants* and the *pattern of repetition of non-logical vocabulary*.
- How can propositions *contain* logical constants or patterns of repetition?
- Sentences \( P \rightarrow Q \) and \( \neg P \lor Q \) seem to have different logical forms.
- But it is not clear that they express different propositions.
- See Dale’s ‘Logical Equivalents and Logical Form’ (1982) and Grandy’s ‘Some Remarks About Logical Form’ (1974) for further reasons.
Propositions

▶ Logical consequence should be language independent (see e.g. Cargile’s ‘Logical Form’, 2010).
▶ Also consider Kripke’s ‘A Puzzle about Belief’ (1979):

Pierre is a native French speaker who has never been to London but believes it to be pretty. He is inclined to utter the sentence ‘Londres est jolie’. After some interaction with English speakers who do not believe London to be pretty, Pierre is inclined to utter the sentence ‘London is not pretty’. He has not realised that Londres is London and has incompatible beliefs.

▶ There is no immediate inconsistency in the sentences ‘Londres est jolie’ and ‘London is not pretty’.
▶ What are inconsistent are the propositions expressed.
Form bearers

- Let’s proceed with interpreted sentences as form bearers.
- In fact, though there is a large debate about form bearers, it can seem rather inconsequential.
- There are few points made in this lecture course that could not be translated into proposition-talk quite correctly.
- What is more consequential is the debate about what forms are.
- It is to this question we now turn.
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- On the question of form-bearers, there were two main candidates; one linguistic, one worldly.
- Similarly on the question of forms themselves:
  - **Schema**: A metalinguistic sentence consisting of the use of (free) metalinguistic schematic variables and the mention of vocabulary from the object language. E.g. $\Phi \land \Psi$
  - **Property**: A worldly entity expressed using a schema.
To talk about forms, it is convenient to use schemata. For example, Lemmon, in *Beginning Logic*:

*If we compare the two arguments* (7) *Tweety is a robin; no robins are migrants; therefore Tweety is not a migrant,* and (8) *Oxygen is an element; no elements are molecular; therefore oxygen is not molecular,* both of which are sound (one drawn from ornithology, the other from chemistry), it is hard to escape the feeling that they have something in common. This something is called by logicians their logical form ... We may state the common pattern of (7) and (8) as follows: (9) m has F; nothing with F has G; therefore m does not have G. Form can thus be studied independently of subject-matter, and ... it is forms of argument, rather than actual arguments themselves, that logic investigates. (1971: 4)
The schematic fallacy

- We use schemata to talk about forms.
- But, Smiley reminds us in ‘The Schematic Fallacy’ (1982), we must not mistake the medium for the message.
- Take the argument:
  ‘Jane is a philosopher, all philosophers drink beer; so Jane drinks beer.’
- We may say this is of either of the forms:
  1. \( a \) is \( F \), everything that is \( F \) is \( G \); so \( a \) is \( G \)
  2. \( \mu \) is \( \Phi \), everything that is \( \Phi \) is \( \Psi \); so \( \mu \) is \( \Psi \)
- There’s nothing to choose between these two, so neither is to be identified with the form.
- The decision would be arbitrary!
Mistakes

- But things get worse: accepting schemata as logical forms leads to logical mistakes.
- Let’s restrict our attention once again to TFL.
- Recall that
  1. a form all of whose instances are valid is a purely valid form.
  2. a form all of whose instances are invalid is a purely invalid form.
  3. a form some of whose instances are valid and some of whose instances are invalid is an impure form.
- Are there any pure forms?
- The answer depends on what we accept as an instance of a form.
Consider the restriction:

**Uniformity** A metalinguistic variable must be replaced uniformly.

With Uniformity as the only restriction, we have *some* purely valid forms, e.g. \( A \therefore A \).

With Uniformity, *all* valid arguments have a purely valid form.

There are also *some* purely invalid forms, e.g. \( A \lor \neg A \therefore B \land \neg B \).
But there will be few purely invalid forms.

- \( A \therefore B \) has valid instance \( P \land \neg P \therefore Q \).

- \( \neg A \therefore B \) has valid instance \( \neg(P \lor \neg P) \therefore Q \).

- \( A \land B \therefore C \) has valid instance \( (P \land \neg P) \land Q \therefore R \).

- \( A \rightarrow B \therefore C \) has valid instance \( (P \lor \neg P) \rightarrow (Q \land \neg Q) \therefore R \).
Distinctness

Consider the further restrictions:

- **Distinctness**: Distinct variables must be replaced by distinct sentences.
- **Atomicity**: Atomic variables must be replaced by atomic sentences.

- With Uniformity, Distinctness and Atomicity all in play, every invalid argument has a purely invalid form.
- Indeed, with all three restrictions in play, every argument has a unique form.
- If that argument is valid, it is a purely valid form.
- If that argument is invalid, it is a purely invalid form.
- There are no impure forms.
Consider Copi:

An invalid argument form is one which has at least one substitution instance with true premises and false conclusion. ... Any argument form is valid which is not invalid; a valid argument form is one which has no substitution instance with true premises and false conclusion. (1954: 21)

We can now see that the truth of claims such as this – common in textbooks – depend on the restrictions in place.

If we read Copi’s ‘valid’ forms as ‘purely valid’ and ’invalid’ as ‘purely invalid’, then he is only correct if he has Uniformity, Distinctness and Atomicity all in play.

Otherwise, impure forms render the dichotomy false.
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- Schemata are *linguistic*, so the schemata we can discuss are limited by the expressive power of the language.
- Smiley writes that ‘the schemata of textbooks are constitutionally incapable of expressing the necessary restrictions’ (1982: 8).
- Why? Take Distinctness: can’t we just add a clause to the effect that $A \neq B$. E.g. $P \therefore Q$ has the form $A \therefore B$, where $A \neq B$?
- No: the identity sign should be flanked by *names of sentences*; not *sentences*.
- The result of the substitution, $P \neq Q$, is ill-formed.
The Schematic Fallacy

Consider the following tautologies of propositional logic:

1. \((P \leftrightarrow Q) \rightarrow (\neg P \leftrightarrow \neg Q)\)
2. \((P \leftrightarrow Q) \rightarrow (P \land R \leftrightarrow Q \land R)\)
3. \((P \leftrightarrow Q) \rightarrow (P \lor R \leftrightarrow Q \lor R)\)

(1), (2) and (3) all have some form in common.

But it cannot be expressed by any basic schema.

We could enrich the notation with the predicate \(S(x, y, z)\), which stands for ‘the result of putting \(x\) for \(y\) in \(z\)’.

Then (1), (2) and (3) are all of the common form \((A \leftrightarrow B) \rightarrow (S(A, C, D) \leftrightarrow S(B, C, D))\).

But ‘\(S\)’ is not an expression of the language.
Forms as worldly

- Rather than *identifying* forms with schemata, let’s say that schemata *express* forms, which are worldly rather than linguistic.

- We then avoid the problem of arbitrariness because we can say that $A \land B$ and $\Phi \land \Psi$ express a common form. No need to make an arbitrary choice.

- We avoid the second problem because forms can exist beyond our abilities to talk about them. We are not limited by the expressive capabilities of some language or other.

- What worldly entities are forms?

- Alex Oliver thinks properties. Tim Smiley thinks functions.

- Perhaps there is an interesting indeterminacy in the metaphysics of forms.
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One or many?

Let’s proceed on the assumption that sentences are the form-bearers, that forms are worldly entities of some sort, which schemata express.

What is the upshot of this discussion for the claim that sentences have just one logical form?

Talk of the logical form of implies uniqueness.
In general, objects can have many forms.

Strawson’s example (from *Introduction to Logical Theory*): a sonnet can have many forms: it can be Shakespearean, or Petrarchan, or Petrarchan with a sestet rhyming *cdecde*.

Oliver’s example: a vase can have many forms: it can be both a cuboid and a cube.

And, indeed, sentences seem no different. We can use different schemata to abstract away from different parts of the sentence.

E.g. $P \land Q$ has forms expressed by $A \land B$, $A \land Q$, $P \land B$, $AcB$, $PcB$, $AcQ$, $A$, $\Phi \land \Psi$, and so on...
This has all been applied to formal language, in particular TFL.

How about forms in English? Some philosophers identify the logical forms of English sentences with their formalisations, some with a property of their formalisation.

In either case, there seem to be many forms:

- there are as many forms as formalisations (recall the relativity of formalisation)
- the relativity multiplies because the formalisations themselves have many forms
Consider Oliver on formalisation:

There is a peculiar tendency to rotate the horizontal process of formalisation by 90°, i.e. to conceive of the process as vertical – the uncovering or revealing of a deep, real, hidden logical form underlying the surface, apparent, overt original. But there is no point in thinking of things this way, only mystery. It is easy to understand how an archaeological dig may reveal hidden structures, or how a physical chemist may distinguish phenomenal properties of a substance from its underlying electronic configuration. But one cannot peel back an English sentence or reveal another one by inspecting it under a microscope. (2010: 340)
Instrumentalism

- This all suggests that *instrumentalism* about logical form is the most attractive view.
- We can speak about *the* form when a unique becomes salient, but not of *the* form, in general.
- If we believe that logical consequence / validity is a *formal* relation/property, then the relativity about form will carry over to logical consequence.
- The implicational powers of sentences will be relative to all the same factors as the forms.
- See *logical pluralism*.
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Two metaphysical questions: what entities have logical forms; what entities are logical forms?

On the first, there is a linguistic candidate (sentences) and a worldly candidate (propositions).

On the second, there is also a (meta)linguistic candidate (schemata) and worldly candidates (properties or functions).

Smiley calls the position that logical forms are schemata ‘the schematic fallacy’.

The view commits us to arbitrary decisions, and a limitation of logical forms to expressive power.

This all suggests that sentences have many logical forms, not one.

This is conducive to instrumentalism about logical form.