The Measurement Problem

The Dreaded Quantum, 7ii2017, J. Butterfield (jb56) Thanks to **Adam Caulton**

1 Quantum mechanics for multiple systems

Consider two systems. If system 1's possible states are represented by the Hilbert space $\mathcal{H}_1$ and system 2's possible states are represented by the Hilbert space $\mathcal{H}_2$, then quantum mechanics dictates that the possible states of the *joint system*, system 1 + system 2, are represented by the Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2$, the *tensor product* of $\mathcal{H}_1$ and $\mathcal{H}_2$.

If $|\phi\rangle_1$ is a possible state-vector for system 1 and $|\chi\rangle_2$ is a possible state-vector for system 2 (we now label the states according to the system, for clarity), then the state-vector

$$|\phi\rangle_1 \otimes |\chi\rangle_2 \tag{1}$$

represents a possible state-vector for the joint system; i.e. this state lies in $\mathcal{H}_1 \otimes \mathcal{H}_2$. (Note therefore the ambiguity in the symbol '$\otimes$': if flanked by Hilbert spaces, then it yields the tensor product of those Hilbert spaces; if flanked by vectors belonging to two Hilbert spaces, it yields a vector that lies in the tensor product of those Hilbert spaces.) Sometimes (as in e.g. Albert 1992), the symbol '$\otimes$' is dropped when writing down joint states.

According to the standard prescription for representing single-system quantities as joint-system quantities (and the Born rule), the joint state (1) gives the same statistics for any system-1-quantity as $|\phi\rangle_1$ and the same statistics for any system-2-quantity as $|\chi\rangle_2$.

The joint Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2$ is not exhausted by state-vectors of the form (1)—which are called *product states*. In general, they may be of the form

$$\sum_{i=1}^{\dim(\mathcal{H}_1)} \sum_{j=1}^{\dim(\mathcal{H}_2)} c_{ij} |\phi_i\rangle_1 \otimes |\chi_j\rangle_2 \tag{2}$$

where $\sum_{i,j} |c_{ij}|^2 = 1$ (i.e. the joint state is normalized).

Notice that, because of states of the form (2), the tensor product of two Hilbert spaces is much richer than their Cartesian product. (Compare: in classical mechanics, the state space of a joint system is given by the *Cartesian product* of its constituents' state spaces.) States of the form (2) that can't also be represented in the form (1) represent *entanglement* between the two systems, which we'll talk more about in later lectures.

In terms of wave-functions, we may write the joint state (2) as

$$\psi(\mathbf{x}_1, \mathbf{x}_2) = \sum_{i=1}^{\dim(\mathcal{H}_1)} \sum_{j=1}^{\dim(\mathcal{H}_2)} c_{ij} \phi_i(\mathbf{x}_1) \chi_j(\mathbf{x}_2) \tag{3}$$

where $\mathbf{x}_1$ represents the position of system 1 and $\mathbf{x}_2$ represents the position of system 2. Note that now we cannot think of $\psi(\mathbf{x}_1, \mathbf{x}_2)$ as representing anything as simple as a wave in physical space.

2 The measurement problem and Schrödinger's cat

Recall that the two central principles in QM that connect the formalism to the physical world—namely, the projection postulate and the Born rule—make ineliminable reference to "measurements". One (minimal) problem to file under "the measurement problem" is to provide a satisfactory account of *what a measurement is*.

Another, more involved, problem makes the first one all the more urgent. It is that a contradiction seems to follow if we attempt to treat a measurement apparatus, or a measuring *agent*, as a quantum system. Schrödinger made this vivid (and bloodthirsty) with his example of a cat locked in a booby-trapped box. The booby-trap consists of a sample of radioactive material (known after a certain time to have a 50% chance of decaying) and a detector that, if it fires, causes a vial of poison gas to break, killing the cat.

Let us treat the detector, like the radioactive sample, as a quantum system. Then, according to quantum mechanics, the initial joint state of the sample and the detector is

$$|\text{intact}\rangle_s \otimes |\text{ready}\rangle_d. \tag{4}$$

We know that, after a certain time, the state of the sample, considered in isolation, evolves to

$$|\text{intact}\rangle_s \qquad \mapsto \qquad \frac{1}{\sqrt{2}}\left(|\text{intact}\rangle_s + |\text{decayed}\rangle_s\right) \tag{5}$$

Note that the amplitudes give the right probabilities, given the Born rule. We also know that the detector, if it is a good one, ought to evolve according to the rule

$$|\text{intact}\rangle_s \otimes |\text{ready}\rangle_d \qquad \mapsto \qquad |\text{intact}\rangle_s \otimes |\text{ready}\rangle_d \tag{6}$$

$$|\text{decayed}\rangle_s \otimes |\text{ready}\rangle_d \qquad \mapsto \qquad |\text{decayed}\rangle_s \otimes |\text{click}\rangle_d \tag{7}$$

But we also know that the Schrödinger equation, which governs all this evolution, is linear. It follows that the joint state evolves according to

$$|\text{intact}\rangle_s \otimes |\text{ready}\rangle_d \qquad \mapsto \qquad \frac{1}{\sqrt{2}}\left(|\text{intact}\rangle_s + |\text{decayed}\rangle_s\right) \otimes |\text{ready}\rangle_d \tag{8}$$

$$= \qquad \frac{1}{\sqrt{2}}\left(|\text{intact}\rangle_s \otimes |\text{ready}\rangle_d + |\text{decayed}\rangle_s \otimes |\text{ready}\rangle_d\right) \tag{9}$$

$$\mapsto \qquad \frac{1}{\sqrt{2}}\left(|\text{intact}\rangle_s \otimes |\text{ready}\rangle_d + |\text{decayed}\rangle_s \otimes |\text{click}\rangle_d\right). \tag{10}$$

But if we treat the detection event as a measurement, then according to the projection postulate the final state ought to be

$$|\text{intact}\rangle_s \otimes |\text{ready}\rangle_d \qquad \text{or} \qquad |\text{decayed}\rangle_s \otimes |\text{click}\rangle_d \tag{11}$$

with probability $\frac{1}{2}$ each, according to the Born rule. (10) and (11) consequences can't *both* be right—i.e. we can't take the superposition of joint states to represent disjoint possibilities—because there are (complicated!) physical quantities that (according to QM) give different statistics for (10) than they do for (11).

It *may be* tempting to treat as a measurement only what constitutes an act of sentient observation. Schrödinger used the cat to show that this response (suggested by Wigner) won't work. If we treat the sample, the detector, the vial *and* the cat all as quantum systems, then the final state of the total system, as far as you are concerned, should be

$$\frac{1}{\sqrt{2}}\left(|\text{intact}\rangle_s \otimes |\text{ready}\rangle_d \otimes |\text{intact}\rangle_v \otimes |\text{alive}\rangle_c + |\text{decayed}\rangle_s \otimes |\text{click}\rangle_d \otimes |\text{broken}\rangle_v \otimes |\text{dead}\rangle_c\right), \tag{12}$$

whereas the projection postulate and the Born rule together entail that the measurement (performed by the cat!) causes the state to "collapse" into

$$|\text{intact}\rangle_s \otimes |\text{ready}\rangle_d \otimes |\text{intact}\rangle_v \otimes |\text{alive}\rangle_c \quad \text{or} \quad |\text{decayed}\rangle_s \otimes |\text{click}\rangle_d \otimes |\text{broken}\rangle_v \otimes |\text{dead}\rangle_c \tag{13}$$

with probability $\frac{1}{2}$ each. Again, (12) and (13) can't both be right.

3 In Schrödinger's own words

From his great 1935 paper (Section 5: Are the variables really blurred?):

That it is in fact not impossible to express the degree and kind of blurring of all variables in one perfectly clear concept follows at once from the fact that Q.M. as a matter of fact has and uses such an instrument, the so-called wave function or ψ-function, also called system vector. ... That it is an abstract, unintuitive mathematical construct is a scruple that almost always surfaces against new aids to thought and that carries no great message. At all events it is an imagined entity that images the blurring of all variables at every moment just as clearly and faithfully as the classical model does its sharp numerical values. Its equation of motion too, the law of its time variation, so long as the system is left undisturbed, lags not one iota, in clarity and determinacy behind the equations of motion of the classical model.

So the latter could be straight-forwardly replaced by the ψ-function so, long as the blurring is confined to atomic scale, not open to direct control. In fact the function has provided quite intuitive and convenient ideas, for instance the "cloud of negative electricity" around the nucleus, etc.

But serious misgivings arise if one notices that the uncertainty affects macroscopically tangible and visible things, for which the term "blurring" seems simply wrong. The state of a radioactive nucleus is presumably blurred . . . One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following diabolical device (which must be secured against direct interference by the cat): in a Geiger counter there is a tiny bit of radioactive substance, so small, that perhaps in the course of one hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives if meanwhile no atom has decayed. The first atomic decay would have poisoned it. The ψ-function of the entire system would express this by having in it the living and the dead cat (pardon the expression) mixed or smeared out in equal parts.

It is typical of these cases that an indeterminacy originally restricted to the atomic domain becomes transformed in to macroscopic indeterminacy, which can then be resolved by direct observation. That prevents us from so naively accepting as valid a "blurred model" for representing reality. In itself it would not embody anything unclear or contradictory. There is a difference between a shaky or out-of-focus photograph and a snapshot of clouds and fog banks.

Then he is sceptical about "Copenhagen", 'the reigning doctrine'. He has earlier (end of Sec. 2) said ironically: 'I hope later to make clear that the reigning doctrine is born of distress.' *Now he adds:*

In the fourth section we saw that it is not possible smoothly to take over models and to ascribe, to the momentarily unknown or not exactly known variables, nonetheless determinate values, that we simply don't know. In Sect. 5. we saw that the indeterminacy is not even an actual blurring, for there are always cases where an easily executed observation provides the missing knowledge. So what is left? From this very hard dilemma the reigning doctrine rescues itself or us by having recourse to epistemology. We are told that no distinction is to be made between the state of a natural object and what I know about it, or perhaps better, what I can know about it if I go to some trouble. Actually—so they say—there is intrinsically only awareness, observation, measurement. If through them I have procured at a given moment the best knowledge of the state of the physical object that is possibly attainable in accord with natural laws, then any further questioning about the "actual state" is meaningless . . .

4 Further reading

- Albert, David Z. (1992), *Quantum Mechanics and Experience* (Cambridge, MA and London: Harvard UP).
- * Bell, John Stewart, 'Six Possible Worlds of Quantum Mechanics', *Foundations of Physics*, 22, no. 10 (1992): 1201-15. http://doi.org/10.1007/BF01889711. Reprinted in his *Speakable and Unspeakable in Quantum Mechanics* (Cambridge: Cambridge University Press, 1987: 2nd ed. 2004), pp. 181-195.
- Cushing, J. (1998), *Philosophical Concepts in Physics* (Cambridge: CUP), Chs. 20-21.
- Hughes, R. I. G. (1989), *The Structure and Interpretation of Quantum Mechanics* (Cambridge, MA and London: Harvard UP), Chapter 9: pp. 259-295.
- Redhead, Michael (1987), *Incompleteness, Nonlocality and Realism* (Oxford: Clarendon), Ch. 2.
- * Schrödinger, Erwin (1935), 'The Present Situation in Quantum Mechanics: (the "Cat Paradox" paper), translated by John Trimmer, *Proceedings of the American Philosophical Society* **124** (Oct. 10, 1980), pp. 323-338. Stable URL: http://www.jstor.org/stable/986572