# Solving the Measurement Problem? Complementarity and dynamical collapse theories

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This lecture covers two approaches to the measurement problem. Recall that in Bell's taxonomy of 'six possible worlds of quantum mechanics', there were three pairs of views: each with a romantic element, and an unromantic element.

Our first approach is the romantic element in the pair one might call *Endorsing the orthodox formalism*. The corresponding unromantic view is instrumentalism: what Bell (2004: 188) calls 'the purely pragmatic view': associated with the one-liner 'shut up and calculate'. The much-used phrase 'Copenhagen interpretation': (i) is ambiguous between these; and (ii) was invented by Heisenberg (cf. Howard (2004) for the history).

Our second approach is the unromantic element in the pair one might call *Changing the formalism*. This pair says that: despite the orthodox formalism's (especially the Schrödinger equation's *unitary* evolution's) supreme empirical success, it *fails*. The 'collapse of the wave-packet' is real: either (unromantically, i.e. our case) in inanimate nature, in a way that is tuned to recover the empirical success of the orthodox formalism; or (romantically, i.e. not our case), in the mind of the observer. (The subsequent lectures will take up both elements of Bell's third pair: the pilot-wave, and the Everettian (aka: many worlds), interpretations.)

## 1 Complementarity: (?) 'the Copenhagen interpretation'

Birkhoff and von Neumann (1936) expounded the following basic analogy between the state spaces of classical and quantum mechanics, and the associated set of propositions (aka: 'events', 'projections') about the values of physical quantities. We begin with their analogy. It will be evident that:

(i): there is deep analogy between (familiar!) propositional logic (more precisely: its 'intensional' or 'possible world' semantics *a la* Carnap and Lewis), and Birkhoff and von Neumann's 'logic' of classical physics: we begin with this theme; and thus also that

(ii): the formal heart of the complementarity interpretation is the logical innovation of quantum theory: viz. that the algebra/lattice of 'events':

(a) is not *Boolean* (in particular: not *distributive*); but

(b) has Boolean subalgebras/sublattices (that overlap in a delicate way).

The key idea will be that we grasp the non-Boolean whole by the patchwork quilt of its Boolean parts. Or in a cartographic metaphor: the 'atlas' of Boolean 'charts'/parts.

A classical phase space $\Gamma$ supports a Boolean algebra of such propositions, $v(A) \in E$, stating that the value $v(A)$ of the quantity $A$ is in the set $E \subset \mathbb{R}$. Here, the Boolean algebra is of sets (viz. subsets of $\Gamma$) under the operations of intersection, union and set-complement ($\cap, \cup, ^c$). Abstractly, 'Boolean algebra' can be defined as a *lattice* (elements $a, b$ etc. with operations of meet and join: $\wedge, \vee$) that:

(i) is orthocomplemented, i.e. there is an operation $\perp$ with $(a^\perp)^\perp = a$ and $a \leq b \Rightarrow b^\perp \leq a^\perp$;

(ii) obeys the distributive laws:

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \; ; \; a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \; . \tag{1}$$

It is worth drawing the *Hasse diagram* for the simplest Boolean lattices:

the 2-element lattice $\{0,1\}$: which is isomorphic to the power set lattice of a singleton, i.e. $\{\emptyset, \{a\}\}$;

the 4-element Boolean lattice, with elements $\{0, a, a^\perp, 1\}$: which is isomorphic to the power set lattice of a two-element set $\{a, a'\}$, i.e. isomorphic to $\{\emptyset, \{a\}, \{a'\}, \{a, a'\}\}$

the 8-element Boolean lattice, with elements $\{0, a, b, c, a^\perp \equiv b \vee c, b^\perp \equiv a \vee c, c^\perp \equiv a \vee b, 1\}$: which is isomorphic to the power set lattice of a three-element set $\{a, b, c\}$.

... And so on ... you surmise that maybe every Boolean lattice is isomorphic to a power set, ordered by set-inclusion. Indeed so, for finite Boolean lattices: (and there is an infinite generalization: the famous *Stone representation theorem for Boolean algebras*).

Given a quantity $A$, e.g. energy, on a classical phase space $\Gamma$, represented as usual by a function $A : \Gamma \to \mathbb{R}$; and given a partition $\{E_i\}$ of the real line, i.e. $E_i \subset \mathbb{R}, E_i \cap E_j = \emptyset$ for $i \neq j$ and $\cup_I \; E_i = \mathbb{R}$: the propositions $v(A) \in E_i$ defines a *Boolean sublattice* of the entire Boolean lattice of propositions about the values of quantities (i.e. lattice of subsets of $\Gamma$).

Thus the vast Boolean lattice/algebra has many Boolean subalgebras/sublattices: they are given by choosing different quantities; or (less interestingly) for a fixed quantity, by choosing different partitions of the real line in which to assert the quantity's values.

When we turn to quantum theory, we will see a vast *non-Boolean* lattice/algebra with many Boolean subalgebras/sublattices, overlapping in a delicate way . . .

A Hilbert space $\mathcal{H}$ supports a lattice $L(\mathcal{H})$ of projections $P$, corresponding to such propositions, $v(A) \in E \subset \mathbb{R}$. Equivalently: it supports a lattice of subspaces, i.e. the projections' ranges. We will speak indifferently of projections and subspaces.

Thus we define: $P \leq Q$ iff the range of $P$ is a subspace of the range of $Q$. This implies that the set of projections (subspaces) forms a lattice with meet, join and and orthocomplement, being:

$P \wedge Q =$ the projector onto $\mathrm{ran}(P) \cap \mathrm{ran}(Q)$;

$P \vee Q =$ the projector onto $\mathrm{ran}(P) + \mathrm{ran}(Q)$; and

the usual orthocomplement $P \mapsto (1 - P)$ is the orthocomplementation in the lattice.

Note the analogy with—and differences from—the set-theoretic operations of intersection, union and complement.

But this lattice is *non*-Boolean .The first distributive law fails, even for the (real) Hilbert space $\mathbb{R}^2$: just take $a, b, c$ as three rays in the plane $\mathbb{R}^2$, so that $b \vee c = 1$ ($\equiv \mathbb{R}^2$) and so lhs $= a$, while rhs $= 0 \vee 0 = 0$.

$L(\mathcal{H})$ has Boolean *sublattices*. The simplest case is when $\mathcal{H}$ has dimension 2. (Again, we can consider the real case, for simplicity.) And pair of orthogonal rays $a, a^\perp$, defines the 4-element Boolean sublattice (cf. above). All these Boolean sublattices have a common 0 and a common 1: but they otherwise do not overlap. *Draw* the Hasse diagram ...

The general situation, for any $L(\mathcal{H})$, is that any family of mutually orthogonal subspaces that is complete (i.e. that sum to $\mathcal{H}$) defines a Boolean sublattice: i.e. $S_i \perp S_j$ for $i \neq j$, and $\oplus S_i = \mathcal{H}$. The subspaces $S_i$ are the atoms of the sublattice.

Equivalently: any family of mutually orthogonal projections that is complete (i.e. that sum to $I$, the identity operator) defines a Boolean sublattice.

Even in $\mathbb{R}^3$, we can see intricate overlapping of the Boolean sublattices. Thus any trio of orthogonal rays passing through the origin in $\mathbb{R}^3$ defines a Boolean sublattice of $L(\mathbb{R}^3)$. Take a trio, with one ray along the $z$-axis (through the 'North pole' and 'South poles'): it defines a Boolean sublattice, $B$ say. Rotating the trio about that axis, we get new rays lying in equatorial plane, skew to the original equatorial rays. This new trio also defines a Boolean sublattice, $B'$ say. $B$ and $B'$ have in common the $z$-axis ray (and its orthocomplement) ... *Draw* the sphere—and the Hasse diagram ...

This intricate overlapping of the Boolean sublattices when the dimension is greater than 2 is the basis of the *Kochen Specker theorem.*

Indeed, in *any* lattice $L$, the intersection of a family $L_i$ of sublattices is a lattice; and so for any set $S$ of elements of a lattice $L$, $S \subset L$, we define the lattice *generated* by $S$ as the smallest lattice containing $S$, i.e. the intersection of all the sublattices containing $S$.

For an orthocomplemented lattice, we say the elements of $S$ are *compatible* iff the lattice generated by $S$ is Boolean. In particular, for a set of two elements, $S \equiv \{a, b\}$, we write $a \leftrightarrow b$ to indicate that $a, b$ are compatible.

$a \leftrightarrow b$ is equivalent to:

$$a = (a \wedge b) \vee (a \wedge b^{\perp}) \; ; \; b = (b \wedge a) \vee (b \wedge a^{\perp}) \tag{2}$$

(Think of two planes, through the origin, in $\mathbb{R}^3$, with the angle between them a right angle.) Similarly: $a \leftrightarrow b$ is equivalent to the existence of three elements $a_1, b_1, c$ that are:

pairwise orthogonal (where $a_1, b_1$ being *orthogonal* means that $a_1 < b_1^{\perp}$: equivalently $b_1 < a_1^{\perp}$), and also obey:

$a = a_1 \vee c$ and $b = b_1 \vee c$.

For projectors $P, Q$ in $L(\mathcal{H})$, compatibility as thus defined is equivalent to commutation in the usual sense of projector algebra. That is: $PQ = QP$; and so it is equivalent to $P, Q$ having a simultaneous eigenbasis, and to $P, Q$ being co-measurable in the usual operational 'projection postulate' sense.

So much for mathematics: now back to interpretation/philosophy ...
Bohr was inspired by Kant—particularly by Kant's idea that the human mind was constrained in understanding the physical world by a particular conceptual framework. Bohr identified this framework with the concepts of classical physics: namely, position, momentum, energy, angular momentum, etc. Thus, for Bohr, to conceive of a (quantum) system as a part of objective reality *is* just to describe it in terms of classical concepts.

But in quantum theory we find that classical concepts (or if you prefer: their quantum cousins) can be incompatible: namely when the self-adjoint operators that represent the quantities do not commute/cannot be measured simultaneously (cf. also Heisenberg's uncertainty principle). Bohr's response was to suggest that classical concepts group together in compatible families, and that any one family may be used to describe any given system; but no two families may be used *together* to describe the same system. (Crucial to Bohr's interpretation is some division between system and measuring device, since the measuring device determines which family of classical concepts are in operation. Beware: this probably should not be interpreted verificationistically.) Moreover, the information about a quantum system expressible in these classical terms *exhausts*, according to Bohr, what may be said about the system. This whole doctrine Bohr called *complementarity*.

To (at least!) a good approximation: this notion of family (aka: 'context', or 'experimental arrangement') can be identified with the formal notion of any of the following (essentially equivalent!) notions:

physical quantity, or self-adjoint operator, or maximal family of mutually orthogonal subspaces, or Boolean sublattice of $L(\mathcal{H})$.

# 2 The dynamical reduction programme of Penrose, Ghirardi et al

## 2.1 An outline

The guiding idea is that despite the orthodox formalism's (especially the Schrödinger equation's *unitary* evolution's) supreme empirical success, it is *not* universally valid. A strictly isolated quantum system evolves *non*-unitarily: the 'collapse of the wave-packet' is real dynamical process. But the revision of the Schrödinger equation is 'mild and subtle', i.e. is tuned to recover the empirical success of the orthodox formalism. Thus there is no fundamental appeal to the notion of "measurement", nor in particular to the projection postulate, so as to specify under what circumstances the 'collapse' takes place. But the revision of the Schrödinger equation is so designed that when we apply it to describe a process that we think of, heuristically, as a measurement, it yields a collapse onto an appropriate post-measurement state, which is very similar to what the projection postulate dictates: and it does so with probabilities for the various alternatives that are very close to (experimentally indistinguishable from!) the orthodox Born-rule probabilities. collapse is to be construed as a dynamical process. In consequence, they deny that the Schrödinger equation has universal validity. In other words, they claim that a *different* theory, close to QM but one that represents state collapse as a dynamical process, is true.

There have long been various such proposals, sometimes (e.g. Penrose) invoking gravity as the 'trigger' for the collapse. The best-known proposal is by Ghirardi, Rimini and Weber ('GRW': often with Pearle). We sketch their first non-relativistic model (1986). They start from the assumption that all physical objects are composed of elementary particles. They then postulate that every elementary particle has, at all times, an intrinsic disposition to spontaneously and randomly alter its state (i.e. "collapse"). This disposition may be quantified by an average collapse-per-unit-time, which is postulated as a new universal constant of nature. (GRW suggest about $10^{-8}$ spontaneous collapses per year.)

Upon collapse, the state of any system becomes strongly "localized"; i.e. its wave-function becomes highly concentrated around some location. (There is a well-defined prescription for this, leading to the postulation of a new universal constant: in effect, it is the width of a just-collapsed wave-function.) The probability that, given a spontaneous collapse, the resulting wave-function is concentrated around any given location is given by the Born rule. (This is the main 'tactic' by which evidential support for QM becomes evidential support for GRW.)

The key point of the GRW proposal is that, if a collapse occurs for any particle, then it occurs also for any other system(s) entangled with it. Hence macroscopic objects, which will typically evolve into entangled states involving an *enormous* number ($\sim 10^{23}$!) of particles, are *fantastically likely* to experience collapse, *fantastically often.* For a very small system, collapse hardly happens, and regular (!) quantum behaviour is free to occur—unless its behaviour is amplified, by a (macroscopic) measuring device. As Bell quips: *the cat is only alive and dead for a split second.*

### 2.2 Problems for GRW

(1) *Why privilege position?* The success of the original GRW model relies on *all* measurements, somewhere along the line, resolving entangled superpositions of eigenstates of *position.* But it seems that not all experiments are like this (e.g. Albert (1992, Ch. 5)). Later work by GRW et al changes the 'preferred quantity' (i.e. the quantity onto approximate eigenstates of which the dynamics drives large enough systems) from position, to a *mass-density.* This choice certainly meets the objection: but having an *ab initio* favoured quantity is still a kind of 'hostage to fortune'.

(2) *The "problem of tails".* Given the uncertainty principle, each post-collapse wave-function must have *some* spread in position, so as to avoid infinite spread in momentum (and therefore energy). (In fact, the collapse's tendency to confine the state in position does *heat* an isolated system: but not observably—yet!) But then GRW have failed to recover truly definite macro-states. How are the "tails" of the wave-function to be interpreted?

(3) *Absolute simultaneity.* It is very hard to adapt these ideas so as to make a model that is fundamentally relativistic (Lorentz-invariant). In a nutshell: considerations of distant entanglements (see later!) suggest that collapse must be *instantaneous* across arbitrarily large distances. This requires a privileged "collapse frame", in apparent conflict with special relativity.

Within the physics community, this is probably considered the worst problem—-as it is also for the pilot-wave theory ...

## 3 Further reading


- Albert, David Z. (1992), *Quantum Mechanics and Experience* (Cambridge, MA and London: Harvard UP), Chs. 4 & 5.
- Bell, J. S. (1987), 'Are There Quantum Jumps?', in *Speakable and Unspeakable in Quantum Mechanics* (Cambridge: CUP), Ch. 22.
- Cushing, J. (1998), *Philosophical Concepts in Physics* (Cambridge: CUP), Chs. 20-21.
- Ghirardi, G. C., Rimini, A., and Weber, T. (1986), 'Unified dynamics for microscopic and macroscopic systems', *Physical Review D* **34**, pp. 470.
- Howard, D. (2004), 'Who Invented the Copenhagen Interpretation?' *Philosophy of Science* **71**, 669682.

- Hughes, R. I. G. (1989), *The Structure and Interpretation of Quantum Mechanics* (Cambridge, MA and London: Harvard UP), Chapter 7 p 178f.; and 10 (especially Section 10.3f.: p. 306f: 'Copenhagen' only).

- Redhead, Michael (1987), *Incompleteness, Nonlocality and Realism* (Oxford: Clarendon), §§1.1-1.5 (pp. 1-30); Section 7.2-7.4 (pp. 155-164), and Appendix III (176-178).