1A Logic — 2014 Model Answers

Section A

1. Attempt both parts of this question.

(a) Each of the following claims is either true or false. For each true claim , show that it is true by providing a suitable formal proof. For each false claim, show that it is false by providing a suitable truth-table: [15 each]

(i) $\vdash ((\neg B \to \neg A) \to A) \to A)$

1	$\vert\ \vert\ \underline{A}$	
2	$\vert\ \vert\ \vert\ \underline{((\neg B \to \neg A) \to A)}$	
3	$\vert\ \vert\ \vert\ A$	R, 1
4	$\vert\ \vert\ ((\neg B \to \neg A) \to A) \to A$	→I, 1–3
5	$\vert\ \vert\ \underline{\neg A}$	
6	$\vert\ \vert\ \vert\ \underline{\neg B}$	
7	$\vert\ \vert\ \vert\ \neg A$	R, 5
8	$\vert\ \vert\ \neg B \to \neg A$	→I, 5–7
9	$\vert\ \vert\ \vert\ \underline{(\neg B \to \neg A) \to A}$	
10	$\vert\ \vert\ \vert\ A$	→E, 8–9
11	$\vert\ \vert\ \vert\ \bot$	⊥I, 5, 10
12	$\vert\ \vert\ \neg((\neg B \to \neg A) \to A)$	¬I, 9–11
13	$\vert\ \vert\ \vert\ \underline{(\neg B \to \neg A) \to A}$	
14	$\vert\ \vert\ \vert\ \bot$	⊥I, 12–13
15	$\vert\ \vert\ \vert\ A$	⊥E, 14
16	$\vert\ \vert\ ((\neg B \to \neg A) \to A) \to A$	→I, 13–15
17	$\vert\ ((\neg B \to \neg A) \to A) \to A$	TND, 1–4, 5–16

(ii) $\neg E \to \neg\neg F, \neg G \to \neg E \vdash G \to \neg F$

The following valuation will produce the required line of the truth table. Let $v(G) = v(F) = v(E) = \mathbf{T}$. Since the antecedents of both sentences to the left of the turnstile are false, both sentences to the left of the turnstile are true. However the antecedent of the sentence to the right of the turnstile is true, and the consequent false, hence the sentence to the right of the turnstile is false.

(iii) $(C \vee \neg A) \to (B \wedge D), C \to \neg B, \neg A \to \neg D \vdash A \wedge \neg C$

1	$(C \vee \neg A) \to (B \wedge D)$	
2	$C \to \neg B$	
3	$\neg A \to \neg D$	
4	$\quad C$	
5	$\quad C \vee \neg A$	$\vee$I, 4
6	$\quad B \wedge D$	$\to$E, 1, 5
7	$\quad B$	$\wedge$E, 6
8	$\quad \neg B$	$\to$E, 2, 4
9	$\quad \bot$	$\bot$I, 7–8
10	$\quad A \wedge \neg C$	$\bot$E, 9
11	$\quad \neg C$	
12	$\quad\quad A$	
13	$\quad\quad A \wedge \neg C$	$\wedge$I, 11–12
14	$\quad\quad \neg A$	
15	$\quad\quad C \vee \neg A$	$\vee$I, 14
16	$\quad\quad B \wedge D$	$\to$E, 1, 15
17	$\quad\quad D$	$\wedge$E, 16
18	$\quad\quad \neg D$	$\to$E, 3, 14
19	$\quad\quad \bot$	$\bot$I, 17–18
20	$\quad\quad A \wedge \neg C$	$\bot$E, 19
21	$\quad A \wedge \neg C$	TND, 12–13, 14–20
22	$A \wedge \neg C$	TND, 4–10, 11–21

(iv) $\neg(T \to S), (S \leftrightarrow (T \leftrightarrow U)) \vdash S \land \neg(T \leftrightarrow U)$

The following valuation function will give us the required line of the truth table: $v(T) = \mathbf{T}, v(S) = v(U) = \mathbf{F}$. Our first sentence to the left of the turnstile is true, since the conditional $(T \to S)$ is false. Our second premise to the left of the turnstile is also true; $T \leftrightarrow U$ is false, as is S, making their biconditional true. However, since S is false, so is the sentence to the right of the turnstile.

(v) $C \leftrightarrow B, B \lor D, (C \lor D) \to A \vdash A$

1	$C \leftrightarrow B$	
2	$B \lor D$	
3	$(C \lor D) \to A)$	
4	$\quad$ B	
5	$\quad$ C	$\leftrightarrow$E, 1, 4
6	$\quad$ $C \lor D$	$\lor$I, 5
7	$\quad$ A	$\to$E, 3, 6
8	$\quad$ $\neg B$	
9	$\quad$ D	DS, 2, 8
10	$\quad$ $C \lor D$	$\lor$I, 9
11	$\quad$ A	$\to$E, 3, 10
12	A	TND, 4–7, 8–11

(vi) $P \leftrightarrow \neg Q, Q \leftrightarrow \neg R \vdash P \leftrightarrow R$

1	$P \leftrightarrow \neg Q$	
2	$Q \leftrightarrow \neg R$	
3	$\quad P$	
4	$\quad \neg Q$	$\leftrightarrow$E, 1, 3
5	$\quad\quad \neg R$	
6	$\quad\quad Q$	$\leftrightarrow$E, 2, 5
7	$\quad\quad \bot$	$\bot$I, 4, 6
8	$\quad \neg\neg R$	$\neg$I, 5–7
9	$\quad R$	DNE, 8
10	$\quad R$	
11	$\quad\quad Q$	
12	$\quad\quad \neg R$	$\leftrightarrow$E, 2, 11
13	$\quad\quad \bot$	$\bot$I, 10, 12
14	$\quad \neg Q$	$\neg$I, 11–13
15	$\quad P$	$\leftrightarrow$E, 1, 14
16	$P \leftrightarrow R$	$\leftrightarrow$I, 2–9, 10–15

(b) Explain the difference between the meanings of ‘$\vdash$’ and ‘$\models$’. Explain, with reference to one of the false claims in part (a), why we are licensed in inferring it's falsity from the truth table that you provided. [10]

The central distinction is the ‘$\vdash$’ is a proof-theoretic symbol, and ‘$\models$’ is model–theoretic, or semantic. More specifically, if $\Gamma \vdash \phi$, that means that there is a proof of ϕ using our natural deduction system the premises of which are all among Γ. By contrast, $\Gamma \models \phi$ means that any valuation function that assigns truth values to the atomic sentences appearing in Γ such that the truth-table for each γ amongst Γ reads ‘**T**‘ is also such that the truth table for ϕ reads ‘**T**’. However, TFL's deductive system is *sound* with respect to its semantics, i.e. if $\Gamma \vdash \phi$, then $\Gamma \models \phi$. Contraposing, we have that if $\Gamma \not\models \phi$ then $\Gamma \nvdash \phi$. Hence, if we can construct a valuation giving us a truth-table under which all the γ's amongst Γ are true and ϕ is false, we know that there is no natural deduction from Γ to ϕ, justifying our conclusions in part (a) numbers (ii) and (iv).

2. Attempt all parts of this question.

(a) Using the following symbolisation key:

Domain: all people
Nx: x is a ninja
Bxy: x is behind y
Sxy: x can see y
a: Akira

symbolise each of the following sentences as best you can in FOL. Comment on your translations where appropriate, in particular highlighting any difficulties in symbolisation. [60]

(i) Everybody behind Akira is a ninja.

$$\forall x(Bxa \to Nx)$$

(ii) If Akira cannot see someone, that person doesn't exist.

$$\neg\exists x\neg Sax$$

Comment: This is slightly awkward because we do not have a predicate expressing existence. Rather, existence is represented in FOL by the existential quantifier. So we cannot formulate an FOL sentence that says of something that Akira cannot see that it is non-existent. Rather we must content ourself with saying that there is no one Akira cannot see.

(iii) Only a ninja standing behind Akira is invisible to him.

$$\forall x(\neg Sax \to (Nx \wedge Bxa))$$

(iv) Akira is invisible to everyone except ninjas.

$$\forall x(\neg Nx \to \neg Sxa)$$

(v) If one ninja stands behind another, the latter can see the former.

$$\forall x\forall y((Nx \wedge Ny) \to Bxy \to Syx)$$

Comment: This is slightly ambiguous between at 'at least one' and 'exactly one'. It seems, however, that the English is more naturally read as meaning that ninjas can see other ninjas that are behind them, rather than as meaning that if exactly one ninja stands behind another, the latter can see the former.

(vi) The ninja that Akira cannot see is behind him.

$\exists x \forall y((Ny \wedge \neg Say) \leftrightarrow x = y) \wedge Bxa)$

(vii) No one can see the ninja behind Akira, not even that ninja herself.

$\exists x \forall y((Ny \wedge Bya) \leftrightarrow x = y) \wedge \neg \exists z Szx)$

Comment: There is no way to emphasise the presumably surprising fact that the ninja behind Akira is so well–concealed that she cannot see herself. Once we have said that *no one* can see her, that is all to be said on the matter as far as FOL is concerned. We could add a clause such as $\wedge \neg Sxx$, but from a logical point of view, this would be superfluous.

(viii) Behind every ninja Akira cannot see, there is another ninja that Akira cannot see.

$\forall x((Nx \wedge \neg Sax) \rightarrow \exists y((Ny \wedge Byx) \wedge \neg Say))$

Comment: There is some temptation here to add a clause to the effect that the second ninja is distinct from the first. However, we needn't do this if we assume (quite reasonably) that nothing is behind itself.

(ix) For every ninja that Akira cannot see, there are two more ninjas that Akira cannot see.

$\forall x((Nx \wedge \neg Sax) \rightarrow \exists y \exists z((((\neg x = y \wedge \neg x = z) \wedge \neg y = z) \wedge (Ny \wedge Nz)) \wedge (\neg Say \wedge \neg Saz)))$

(x) Akira can see each of the three ninjas.

$\exists x \exists y \exists z \forall v(((Nv \leftrightarrow ((v = x \vee v = y) \vee v = z)) \wedge ((\neg x = y \wedge \neg x = z) \wedge \neg y = z)) \wedge ((Sax \wedge Say) \wedge Saz))$

(b) Formalise these arguments, and then use natural deduction to show that they are valid: [40]

(i) If Akira cannot see someone, that person doesn't exist. The ninja is behind Akira. So Akira can see the ninja.

$$\neg\exists x\neg Sax, \exists x\forall y((Ny \leftrightarrow x = y) \wedge Bax) \therefore \exists x\forall y((Ny \leftrightarrow x = y) \wedge Sax)$$

1	$\neg\exists x\neg Sax$	
2	$\exists x\forall y((Ny \leftrightarrow x = y) \wedge Bax)$	
3	$\quad \forall y((Ny \leftrightarrow b = y) \wedge Bab)$	
4	$\quad (Nc \leftrightarrow b = c) \wedge Bab)$	$\forall$E, 3
5	$\quad (Nc \leftrightarrow b = c)$	$\wedge$E, 4
6	$\quad\quad \neg Sab$	
7	$\quad\quad \exists x\neg Sax$	$\exists$I, 6
8	$\quad\quad \bot$	$\bot$I, 1, 7
9	$\quad \neg\neg Sab$	$\neg$I, 6–8
10	$\quad Sab$	DNE, 9
11	$\quad (Nc \leftrightarrow b = c) \wedge Sab$	$\wedge$I, 5, 10
12	$\quad \forall y((Ny \leftrightarrow b = y) \wedge Sab)$	$\forall$I, 11
13	$\quad \exists x\forall y((Ny \leftrightarrow x = y) \wedge Sax)$	$\exists$I, 12
14	$\exists x\forall y((Ny \leftrightarrow x = y) \wedge Sax)$	$\exists$E, 2–13

(ii) There is at least one ninja whom Akira cannot see. Behind every ninja Akira cannot see, there is another ninja that Akira cannot see. So there are at least two ninjas that Akira cannot see.

$\exists x(Nx \wedge \neg Sax), \forall x((Nx \wedge \neg Sax) \rightarrow \exists y((Ny \wedge \neg Say) \wedge \neg x = y)) \therefore \exists x \exists y(((Nx \wedge Ny) \wedge (\neg Sax \wedge \neg Say)) \wedge \neg x = y)$

1	$\exists x(Nx \wedge \neg Sax)$	
2	$\forall x(((Nx \wedge \neg Sax) \rightarrow \exists y((Ny \wedge \neg Say) \wedge \neg x = y))$	
3	$\quad \vert\ Nb \wedge \neg Sab$	
4	$\quad \vert\ ((Nb \wedge \neg Sab) \rightarrow \exists y((Ny \wedge \neg Say) \wedge \neg b = y))$	∀E, 2
5	$\quad \vert\ \exists y((Ny \wedge \neg Say) \wedge \neg b = y)$	→E, 3, 4
6	$\quad \vert \quad \vert\ ((Nc \wedge \neg Sac) \wedge \neg b = c)$	
7	$\quad \vert \quad \vert\ Nc \wedge \neg Sac$	∧E, 6
8	$\quad \vert \quad \vert\ \neg b = c$	∧E, 6
9	$\quad \vert \quad \vert\ Nb$	∧E, 3
10	$\quad \vert \quad \vert\ \neg Sab$	∧E, 3
11	$\quad \vert \quad \vert\ Nc$	∧E, 7
12	$\quad \vert \quad \vert\ \neg Sac$	∧E, 7
13	$\quad \vert \quad \vert\ Nb \wedge Nc$	∧I, 9, 11
14	$\quad \vert \quad \vert\ \neg Sab \wedge \neg Sac$	∧I, 10, 12
15	$\quad \vert \quad \vert\ ((Nb \wedge Nc) \wedge (\neg Sab \wedge \neg Sac))$	∧I, 13, 14
16	$\quad \vert \quad \vert\ (((Nb \wedge Nc) \wedge (\neg Sab \wedge \neg Sac)) \wedge \neg b = c)$	∧I, 8, 15
17	$\quad \vert \quad \vert\ \exists y(((Nb \wedge Ny) \wedge (\neg Sab \wedge \neg Say)) \wedge \neg b = y)$	∃I, 16
18	$\quad \vert\ \exists y(((Nb \wedge Ny) \wedge (\neg Sab \wedge \neg Say)) \wedge \neg b = y)$	∃E, 6–17
19	$\quad \vert\ \exists x \exists y(((Nx \wedge Ny) \wedge (\neg Sax \wedge \neg Say)) \wedge \neg x = y)$	∃I, 18
20	$\exists x \exists y(((Nx \wedge Ny) \wedge (\neg Sax \wedge \neg Say)) \wedge \neg x = y)$	∃E, 3–19

3. Attempt all parts of this question.

(a) Define the following set-theoretic notions: union, intersection, subset, power set, Cartesian product. [10]

$A \cup B = \{x | x \in A \vee x \in B\}$
$A \cap B = \{x | x \in A \wedge x \in B\}$
$A \subseteq B \leftrightarrow \forall x(x \in A \rightarrow x \in B)$
$\mathcal{P}(A) = \{x | x \subseteq A\}$
$A \times B = \{(a, b) | a \in A \wedge b \in b\}$

(b) Each of the following statements is either true or false. In each case say which and briefly explain why: [45]

(i) For any sets A, B and C, if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

True. By hypothesis, all members of A are members of B. All members of B, including the members of A, are in C. Hence, all the members of A are members of C, i.e. $A \subseteq C$.

(ii) For any sets A and B, if $A \subseteq B$ then $A \subseteq \mathcal{P}(B)$.

False. This is because A might contain an individual (i.e. non–set) as a member. Yet all members of $\mathcal{P}(B)$ are sets, since it is a set of subsets. E.g. Let $A = B = \{\text{Kanye}\}$. Then $\mathcal{P}(B) = \{\emptyset, \{\text{Kanye}\}\}$. But Kanye $\in A$ and Kanye $\notin \mathcal{P}(B)$, hence $A \not\subseteq \mathcal{P}(B)$.

(iii) For any sets A and B, if $A \in \mathcal{P}(B)$ then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

True. Suppose otherwise. Since $A \in \mathcal{P}(B)$, we have it that $A \subseteq B$. But $\exists a(a \in \mathcal{P}(A) \wedge a \notin \mathcal{P}(B))$, by hypothesis that $\mathcal{P}(A) \not\subseteq \mathcal{P}(B)$. So $a \subseteq A$ and $a \not\subseteq B$. Hence, $\exists x(x \in a \wedge (x \in A \wedge x \notin B))$. But $A \subseteq B$, so $x \in B$. Contradiction.

(iv) For any sets A, B and C, $(A \cap B) \times (B \cap C) \subseteq (A \times C)$.

True. $(A \cap B) \times (B \cap C) = \{(x, y) | (x \in A \wedge x \in B) \wedge (y \in B \wedge y \in C)\}$. If $x \in A \wedge x \in B$, then $x \in A$. Similarly if $y \in B \wedge y \in C$, then $y \in C$. Hence, if $(x, y) \in (A \cap B) \times (B \cap C)$, then $(x, y) \in A \times B$. That is, $(A \cap B) \times (B \cap C) \subseteq (A \times C)$.

(v) For any set A, $\mathcal{P}(A) \in \mathcal{P}(\mathcal{P}(A))$.

True. Since $\mathcal{P}(A) \subseteq \mathcal{P}(A)$, $\mathcal{P}(A) \in \{x | x \subseteq \mathcal{P}(A)\} = \mathcal{P}(\mathcal{P}(A))$.

(c) Define the following notions from the logic of relations: reflexive, symmetric, transitive, equivalence relation. [10]

A relation, R, is reflexive with respect to a given domain if, and only if, $\forall x Rxx$.

A relation, R, is symmetric with respect to a given domain if, and only if, $\forall x \forall y (Rxy \rightarrow Ryx)$.

A relation, R, is transitive with respect to a given domain if, and only if, $\forall x \forall y \forall z ((Rxy \wedge Ryz) \rightarrow Rxz)$.

A relation is an equivalence relation with respect to a given domain if, and only if, it is reflexive, symmetric and transitive with respect to that domain.

(d) Call a relation R negatively transitive if $\forall x \forall y \forall z ((\neg Rxy \wedge \neg Ryz) \rightarrow \neg Rxz)$. Give examples of relations with each of the following properties: [35]

In all cases, we let the domain $= \mathbb{N}$, the natural numbers.

(i) Equivalence, but not negatively transitive.

The identity relation. This is an equivalence relation with respect to any domain. Let $x = z$ and $\neg x = y$. Then $\neg Rxy \wedge \neg Ryz$ but Rxz.

(ii) Reflexive and negatively transitive, but not symmetric.

The relation $\not>$ ('is not greater than') will do the trick. This is reflexive with respect to the domain, since no number is greater than itself. $\neg Rxy$ if and only if $x > y$, which is transitive with respect to the domain, so $\not>$ is negatively transitive. But this relation isn't symmetric, since $0 \not> 1$ but $1 > 0$.

(iii) Reflexive and symmetric, but not transitive.

Let $R = \{(x, y) | x = y \vee x = y + 1 \vee x + 1 = y\}$. This is reflexive since every number is self-identical, and symmetric, since if $x = y + 1$ then $y + 1 = x$ and vice–versa. But it isn't transitive, since $R01$ and $R12$ and $\neg R02$.

(iv) Transitive and symmetric, but not reflexive.

The let $R = \emptyset$. This isn't reflexive, since there are numbers than don't bear the empty relation to themselves. But the conditions for symmetry and transitivity are universal generalisations, which are trivially satisfied by the empty relation.

(v) Transitive, but neither symmetric nor negatively transitive.

Let $R = \{(0, 2)\}$. This relation is somewhat gerrymandered, but it works. R

is transitive; since no counterexample can be constructed as exactly one instance of the relation holds. It isn't symmetric, since $(2, 0) \notin R$, and it isn't negatively transitive either, since $\neg R01$ and $\neg R12$ but $R02$.

4. Attempt all parts of this question.

(a) Define the terms: Field, event space, conditional probability. [10]

$\mathcal{F} = \mathcal{P}(V)$
$V =$ the set of possible outcomes of a given trial.
$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$

(B) You are held captive in the Bayesian Republic of Zembla. The gaoler places two bullets in consecutive chambers in a six-chambered revolver, spins the wheel, and takes a shot at you. You are in luck: the camber was empty! The gaoler then decides he will take a second shot. However, he decides to let you choose between the following options: *either* he will fire from the next chamber, *or* he will spin the chamber again and then fire. Which should you choose and why? [15]

We'll assume that if a round is chambered in a given cylinder, the gaoler will not miss. Label the cylinders of the revolver clockwise 1 through 6. We know that the bullets are in consecutive cylinders, so the set of possible loading configurations $= \{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 5\}, \{5, 6\}, \{6, 1\}\}$. Further assuming that the Gaoler's spin is fair each time, the probability of you getting shot if you choose the gaoler's second option, is $\frac{1}{3}$: Given that you were missed the first time, there are two bullets left in the revolver. There are $6 \times 6 = 36$ possible outcomes (6 possibilities for the chamber fired, and 6 configurations of loading). 12 of these are lethal, since whichever chamber is fired, two of the six loading configurations will result in you getting shot.

We'll proceed by calculating the odds of getting shot after choosing the first option. Assume that the chamber 1 is fired first. Given that you were missed on the first shot, only four loading configurations are compatible with our information, namely that the following chambers of the revolver are loaded: $\{2, 3\}, \{3, 4\}, \{4, 5\}, \{5, 6\}$. In one of these outcomes, taking the gaoler's first option will result in you being shot, namely if chambers 2 and 3 are loaded. Likewise for the scenarios in which each of the other five chambers is emptied first. So there are 24 possible outcomes, of which 6 are lethal. In other words, the probability of getting shot on the gaoler's first option is $\frac{1}{4}$.

As $\frac{1}{4} < \frac{1}{3}$, if you don't want to get shot, you should take the second option.

(c) One octopus in every 100 is psychic. Psychic octopoi have perfect knowledge of the results of future football tournaments. Non-psychic octopoi can only guess randomly. Paul, a randomly chosen octopus, correctly predicts the winner of the next (football) World Cup, from the 32 teams that qualified. What is the probability that Paul is Psychic? [25]

Let 'H' mean that Paul is psychic. Let 'E' mean that Paul has selected the winning football team. To answer this question, we'll plug the values we've been given into:

Bayes's Theorem: $Pr(H|E) = \frac{(Pr(E|H)Pr(H))}{((PrE|H)Pr(H))+(Pr(E|\neg H)Pr(\neg H))}$

We know that $Pr(E|H) = 1$, since psychic octopoi are right every time; that $Pr(H) = 0.01$ and $Pr(\neg H) = 0.99$, since only one in a hundred octopoi are psychic; and that $Pr(E|\neg H) = \frac{1}{32}$, since non-psychic octopoi pick a team at random. So Bayes's theorem tells us that:

$$Pr(H|E) = \frac{0.01}{0.01+(\frac{1}{32}\times 0.99)} = \frac{0.01}{0.01+\frac{99}{3200}} = \frac{0.01}{(\frac{131}{3200})} \approx 0.24$$

(d) I choose three different numbers, at random, from the (whole) numbers between 1 and 4 (inclusive). What is the probability that my choices are in increasing (not necessarily consecutive numerical order? [15]

Given that we are choosing three times without replacement from an initial selection of four, there are $(4 \times 3 \times 2) = 24$ sequences that we might select. How many of these are in increasing numerical order? There are 0 such sequences the first choice of which is 4 or 3. If the first choice is 2, then the sequence (2, 3, 4) is the only acceptable sequence. If our first pick is 1, then the sequences (1, 2, 3), (1, 2, 4), and (1, 3, 4) are the only acceptable sequences. Therefore, four sequences satisfy the condition of being in increasing numerical order. Assuming that our selection is random and fair, then, the odds of picking such a sequence are $\frac{4}{24} = \frac{1}{6}$.

(e) I choose three different numbers, at random, from the (whole) numbers between 1 and 10 (inclusive). What is the probability that they are in increasing (not necessarily consecutive) numerical order?

In this case there are $(10 \times 9 \times 8) = 720$ combinations that we might pick, so we can't work through the cases manually. However, if the first number in the sequence is n, then there are $10 - n$ possible second places which are larger than our first pick. Similarly, if our second pick is m, there are $10 - m$ possible selections of third place which will satisfy the condition of increasing numerical order. Exploiting the similarity between this relationship and the triangular numbers, we can expect that the number of acceptable sequences starting with n will be equal to the sum of the first $10 - n$ numbers (inclusive of 0). Therefore the sum of the first 10 such sums will be the number of sequences satisfying the condition of increasing numerical order. More formally, our answer equals:

$$\sum_{m=1}^{10} \left(\sum_{n=0}^{(10-m)} n \in \mathbb{N} \right)$$

Working through this, we have:

36 (0+1+2+3+4+5+6+7+8)
+28 (0+1+2+3+4+5+6+7)
+21 (0+1+2+3+4+5+6)
+15 (0+1+2+3+4+5)
+10 (0+1+2+3+4)
+6 (0+1+2+3)
+3 (0+1+2)
+1 (0+1)
+0 (unary sum of 0)
+0 (nullary sum)
= 120

Assuming that our selection was fair then, the odds of picking a numerically increasing sequence are $\frac{120}{720} = \frac{1}{6}$