

**PHILOSOPHY TRIPOS Part IA**

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Tuesday 26 May 2009

09.00 to 12.00

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Paper 3

LOGIC

Answer **three** questions only; at least **one** from **each** section.

*Write the number of the question at the beginning of each answer.*

**STATIONERY REQUIREMENTS**

*20 Page Answer Book x 1*

*Rough Work Pad*

**You may not start to read the questions  
printed on the subsequent pages of this  
question paper until instructed that you  
may do so by the Invigilator**

SECTION A

1 Attempt all parts of this question.

(a) Carefully explain the following:

- (i) truth-function
- (ii) tautology
- (iii) tautological entailment
- (iv) metalanguage

(b) Explain the syntactic rules governing the language PL.

(c) What does it mean to say that the tree method for PL is sound and complete?

(d) What does it mean to say that the set of connectives  $\{\neg, \vee\}$  is expressively adequate? Prove that it is.

2 Attempt all parts of this question.

(a) Using the following translation manual:

- 'a' denotes Alice
- 'b' denotes Bertrand
- 'Lx' expresses: x is a logician
- 'Px' expresses: x is a philosopher
- 'Axy' expresses: x admires y

and taking the domain of quantification to be all people, translate the following sentences into QL= as best you can:

- (i) Only if Bertrand is a philosopher are all logicians philosophers.
- (ii) No philosophers who are not logicians are admired by Bertrand.
- (iii) Every philosopher admires some logician or other.
- (iv) Some philosophers are admired by no logician.
- (v) Any philosopher who admires some logician admires Bertrand.
- (vi) Only if Alice is someone other than Bertrand does some philosopher not admire herself.
- (vii) Only Bertrand admires all those who do not admire themselves.
- (viii) Whoever admires some logician only admires Bertrand.
- (ix) Alice admires the philosopher who admires Bertrand.
- (x) There are exactly two logicians, and at least one of them is a philosopher.

[TURN OVER for continuation of question 2]

- (b) Show that the following arguments are valid by translating them into QL= and using trees.
- (i) Some logicians are not wise. All philosophers are wise. Hence some logicians are not philosophers.
  - (ii) Only Bertrand is a great philosopher. Russell is a great philosopher. Hence Bertrand is none other than Russell.
  - (iii) If Bertrand is a philosopher then some logicians are philosophers. No philosopher is a fool. So if all logicians are fools, Bertrand is not a philosopher.
  - (iv) All logicians are philosophers. So any logician's brain is a philosopher's brain.
  - (v) The one and only person who composed the *Iliad* composed the *Odyssey*. Homer composed the *Iliad*. Hence Homer composed the *Odyssey*.
  - (vi) There is a girl who is admired by all the boys. So all the boys admire some girl or other.

3 Attempt all parts of this question.

- (a) Explain the expressions  $\emptyset$ ,  $X \cup Y$ ,  $X \cap Y$ ,  $X \subseteq Y$ ,  $X - Y$ ,  $\wp(X)$ .
- (b) Let A = the set of all male people and B = the set of all female people. Using 'A' and 'B' and some of the expressions explained in (a), write down set-theoretic expressions for
  - (i) the set of all people
  - (ii) the set of all single-sex sets
  - (iii) the set of all mixed-sex sets
- (c) Using examples, explain the difference between the set-membership and subset relations.
- (d) Using examples, explain (i) equivalence relation (ii) equivalence class.
- (e) A relation  $R$  is *Euclidean* iff  $\forall x \forall y \forall z ((Rxy \wedge Rxz) \supset Ryz)$ . Stating the chosen domain in each case, give examples of relations that are
  - (i) reflexive and symmetric but not transitive
  - (ii) symmetric, transitive and Euclidean but not reflexive
  - (iii) Euclidean but neither reflexive nor transitive
  - (iv) transitive and reflexive but neither Euclidean nor symmetric

4 Attempt all parts of this question.

- (a) Define the conditional probability  $P(A/B)$ .
- (b) State Bayes's Theorem.

[TURN OVER for continuation of question 4]

- (c) A fair die is thrown twice. Calculate the probability that
- (i) the first throw is a 5
  - (ii) the first throw is neither a 6 nor a 5
  - (iii) the second throw is even but not a 4
  - (iv) both throws are even given that at least one is even
  - (v) both throws are even given that at least one is a 4
- (d) A new test for a disease gives a positive reading for 98% of those people who have it, and a positive reading for 1% of those who do not. Suppose you live in a population of whom 0.1% have the disease. You visit the doctor and the test gives a positive reading. What is the probability that you have the disease?

## SECTION B

- 5 What are the constituents of arguments?
- 6 Is 'The present King of France is bald' plainly false? Is 'The present King of France does not exist' plainly true? Explain the significance of your answers for the analysis of definite descriptions.
- 7 Are logical truths necessarily true? If so, why? If not, why not?
- 8 In what sense, if any, are the paradoxes of material implication genuine paradoxes?
- 9 Is the verification principle true or false or neither?

END OF PAPER