

**PHILOSOPHY TRIPOS Part IA**

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Tuesday 31 May 2011

09.00 to 12.00

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Paper 3

LOGIC

*Answer **three** questions only; at least one from each section.*

*Write the number of the question at the beginning of each answer.*

**STATIONERY REQUIREMENTS**

*20 Page Answer Book x 1*

*Rough Work Pad*

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator**

SECTION A

- 1 Attempt all parts of this question.
- (a) Carefully define the following notions:
- (i) truth-function
  - (ii) tautological entailment
  - (iii) material conditional
  - (iv) metalanguage
- (b) Explain, with illustrations, what is meant by saying that a deductive system is sound and complete. Give examples of deductive systems for propositional logic that are:
- (i) sound but not complete
  - (ii) complete but not sound
- (c) What does it mean to say that any truth-function can be expressed using just the connectives  $\wedge$ ,  $\vee$  and  $\neg$ ? Prove this.
- 2 Attempt all parts of this question. Through all parts of this question, take the domain to be all people and use the following translation scheme:
- 'j' denotes Jenny
  - 'k' denotes Karl
  - 'Mx' expresses: x is miserable
  - 'Bx' expresses: x is a Bolshevik
  - 'Lxy' expresses: x loves y
- (a) Translate the following into QL=, commenting on any difficulties.
- (i) Karl loves Jenny if and only if the latter is not miserable.
  - (ii) Anyone who loves no miserable person loves neither Jenny nor Karl.
  - (iii) Jenny is not miserable only if she is the only person loved by Karl.
  - (iv) Precisely two Bolsheviks love Jenny and precisely one of those two is miserable.
  - (v) No one who loves the only miserable person loves the only Bolshevik.
  - (vi) If two Bolsheviks love Karl then they love one another.

**[continuation of question 2 on page 3]**

- (b) Render the following arguments into QL=. Then use trees to show that they are valid.
- (i) Jenny is miserable but she loves exactly those people whom Karl loves. Karl loves everyone who is miserable. Therefore Jenny loves herself.
  - (ii) Karl is the only person who is not miserable, and he is a Bolshevik. Therefore Karl is the only person who is both Bolshevik and not miserable.
  - (iii) Karl is not a Bolshevik only if Jenny is miserable. Jenny is not miserable if someone loves her. Karl loves no one who loves any Bolshevik who loves Jenny. Karl loves only Jenny. Therefore Karl does not love himself.
  - (iv) Everyone who is loved loves everyone. Exactly one person is utterly unloved. Therefore there is exactly one person.

3 Explain what it is for a relation to be reflexive, symmetric and transitive. We say that a relation  $R$  is *negatively transitive* if and only if  $\forall x \forall y \forall z ((\neg Rxy \wedge \neg Ryz) \supset \neg Rxz)$ . Of the following relations say which are reflexive, symmetric, transitive and negatively transitive on the domain of people. In each case give a brief explanation. In questions (a) – (e) assume that all siblings share both parents. In question (i) assume that Jane loves John, and John loves Jane, but nobody else loves either of them.

- (a)  $x$  is  $y$ 's father.
- (b)  $x$  is a brother of  $y$ .
- (c)  $x$  is  $y$ 's only sibling.
- (d)  $x$  and  $y$  have no common ancestor.
- (e)  $x$  is an ancestor of  $y$ .
- (f)  $x$  loves  $y \equiv y$  loves  $x$ .
- (g)  $x$  loves  $y \vee y$  loves  $x$ .
- (h)  $\forall z (x \text{ loves } z \equiv z \text{ loves } y)$ .
- (i)  $x$  loves John  $\equiv y$  loves Jane.
- (j) A majority of people prefer  $x$  to  $y$ .

4 In probability, what is an event space? What is a field? What is conditional probability? What is Bayes's Theorem? Now solve the following problems. In questions (b) and (c) please assume that there are exactly as many boys as girls.

**[TURN OVER for continuation of question 4]**

- (a) Johnny draws two socks at random from a drawer containing six socks, all either black or white. His chance of drawing a pair of white socks is  $\frac{2}{3}$ . What is his chance of drawing a pair of black socks?
- (b) Jane has two children. One is a boy. What is the probability that she has two boys?
- (c) Jane has two children. One is a boy who was born on a Monday. What is the probability that she has two boys? If (b) and (c) have different answers then explain briefly why.
- (d) Coin A has a  $\frac{1}{2}$  chance of landing heads. Coin B has a  $\frac{1}{3}$  chance of landing heads. Billy tosses one of them at random and it lands tails. What is the probability that he has tossed coin A?
- (e) 20% of a certain population takes an illegal drug. A test for this drug gives the correct result 90% of the time whether or not the subject has taken the drug. A random subject tests positive. What is the probability that he has taken the drug?

## SECTION B

- 5 How much of the meaning of 'if' does ' $\supset$ ' capture?
- 6 Does 'The cat is on the moon' mean the same as 'There is one and only one cat and it is on the moon'?
- 7 Can all necessary truths be known a priori?
- 8 Can the verification theory of meaning be given a formulation that is both clear and defensible?

END OF PAPER