Paper 3

LOGIC

*Answer three questions only; at least one from each section.*

*Write the number of the question at the beginning of each answer.*

**STATIONERY REQUIREMENTS**  
20 Page Answer Book x 1  
Rough Work Pad
Each question has equal weight. A perfect answer would receive a notional 100 points. For Part A [formal questions] the number in square brackets after each component of a question designates the number of points that a full and correct answer to that component would merit.

SECTION A

1  (a) Carefully define the following:

(i) validity; [5]
(ii) tautological validity; [5]
(iii) a tautology. [5]

(b) What is the difference between ‘∴’, ‘⊦’, and ‘⊧’? [15]

(c) Can an inference be made invalid by adding extra premises? Give a reason for your answer. [15]

(d) The tree method for PL is sound and complete. Explain carefully what this means. [15]

(e) The connectives {¬, ∨} are expressively adequate. Explain what this means and prove that they are. [40]

2  (a) Carefully define the following terms in the context of the syntax and semantics of QL=:

(i) term; [5]
(ii) operator; [5]
(iii) scope; [10]
(iv) q-valuation; [10]
(v) extended q-valuation; [10]
(vi) q-validity. [10]

(b) Explain the semantics of the QL operators ∀ and ∃. Explain why any inference of the form ¬∃xFx, ⊢ ∀xFx is q-valid. [50]
3 (a) Let $R$ be a relation. State the following conditions on $R$ in the language of $QL =$:

(i) $R$ is reflexive; [5]
(ii) $R$ is symmetric; [5]
(iii) $R$ is transitive. [5]

(b) Let us say that a relation $R$ is Euclidean if $\forall x \forall y \forall z ((Rxy \land Rxz) \supset Ryz)$. Let us say that $R'$ is the converse of $R$ if $\forall x \forall y (Rxy \equiv R'yx)$.

Now translate the following arguments into $QL =$ and show using trees that they are valid.

(i) if $R$ is reflexive and symmetric and transitive then $R$ is Euclidean; [15]
(ii) if $R$ is symmetric and Euclidean and $R'$ is the converse of $R$ then $R'$ is symmetric and Euclidean; [15]
(iii) if $R$ is reflexive and $R'$ is the converse of $R$ then $R'$ is reflexive. [15]

(c) Using the definitions in (b), give examples of relations that are:

(i) Euclidean and symmetric but not transitive; [10]
(ii) transitive and Euclidean but not symmetric; [10]
(iii) transitive and symmetric but not reflexive; [10]
(iv) not Euclidean but with a Euclidean converse. [10]

4 (a) You randomly draw two cards without replacement from a standard pack containing 52 cards: 13 of each suit (spades, hearts, diamonds, clubs) and no jokers. What is the probability that:

(i) the second is a spade? [5]
(ii) the first is a spade given that the second is not a spade? [10]
(iii) the second is a club given that the first is a heart? [10]
(iv) the second is a diamond given that the first is a diamond? [10]
(v) the second is a diamond given that the first is the king of diamonds? [15]
(b) Billy is on trial for a crime. The DNA test shows that somebody with Billy's DNA was at the scene of the crime. 0.01% of the UK population shares Billy's DNA. So, the prosecution argues, there is a 99.99% probability that Billy was at the scene. Not so, says the defence: the UK population is 60 million, so there are 6,000 people with Billy's DNA, so the probability that Billy was at the scene is 1/6000. Who is right? [50]

SECTION B

5 Does 'The largest odd number is greater than 100' have a truth value?

6 Can there be synthetic necessary truths?

7 Explain (a) the type/token distinction and what is meant by 'an indexical expression' and (b) the relevance of these notions to logic.

8 What are the 'paradoxes of material implication'? How would you set about trying to resolve the concerns they might raise?

END OF PAPER