# 1A Logic Model Answers

1.

(a)

(i) A truth-function is a function that takes one or many truth-values as inputs and gives a truth-value as an output.

(ii) A set of sentences $P_1, P_2, \ldots P_n$ tautologically entails a sentence Q if and only if there is no assignment of truth values to the atoms of $P_1, P_2 \ldots P_n$ and Q on which $P_1, P_2, \ldots P_n$ are all true and Q is false.

(iii) The material conditional $\supset$ is the two-place propositional connective such that $P \supset Q$ is true unless P is true and Q is false.

(iv) The metalanguage is the language in which one describes properties and relations of elements of the object language e.g. tautological entailment.

(b)

(i) A system that is sound but not complete is the system that allows nothing to be deduced from anything.

(ii) A system that is complete but not sound is the system that allows anything to be deduced from anything.

(c) To say that a truth-function can be expressed by $\wedge$, $\vee$ and $\neg$ is to say that every truth function is some combination of these truth-functions. To prove it: consider a truth-function G that takes n truth-values to a truth-value. Then G's truth-table is as follows:

| $P_1$ | $P_2$ | … | $P_n$ | G |
|---|---|---|---|---|
| T | T | … | T | $X_1$ |
| F | T | … | T | $X_2$ |
| T | F | … | T | $X_3$ |
| F | F | … | T | $X_4$ |
| … | … | … | … | … |
| … | … | … | … | … |
| F | F | … | F | $X_k$ |

Each of $X_1, X_2, \ldots X_k$ is either T or F. Consider all the rows $j_1, j_2, \ldots j_q$ in which $X_j$ = T. Call each such row a *good* row. Now define for each i:

$H(P_i, m) = P_i$ if $P_i$ is T in row m
$H(P_i, m) = \neg P_i$ if $P_i$ is F in row m
$E(m) = H(P_1, m) \wedge H(P_2, m) \wedge \ldots \wedge H(P_n, m)$

Then $G(P_1, \ldots P_n) = E(j_1) \vee E(j_2) \vee \ldots \vee E(j_q)$ where $j_1, j_2, \ldots j_q$ index all and only the good rows. And clearly $E(j_1) \vee E(j_2) \vee \ldots \vee E(j_q)$ involves only $\wedge$, $\vee$ and $\neg$.

2. (a)

(i) $Lkj \equiv \neg M$

(ii) $\forall x (\neg \exists y Lxy \supset (\neg Lxj \wedge \neg Lxk))$

(iii) $\neg Mj \supset \forall x (Lkx \equiv x = j)$

(iv) $\exists x \exists y \forall z ((((z \neq y \vee z \neq x) \wedge Lzj) \equiv (z = x \vee z = y)) \wedge Mx \equiv \neg My)$

(v) $\exists x \exists y \forall z ((Bz \equiv z = x) \wedge (Mz \equiv z = y) \wedge (Lzy \supset \neg Lzx))$

(vi) $\forall x \forall y ((Bx \wedge By \wedge y \neq x \wedge Lxk \wedge Lyk) \supset (Lxy \wedge Lyx)$

(b) (i)

$$Mj \wedge \forall x(Ljx \equiv Lkx)$$

$$\forall x(Mx \supset Lkx)$$

$$\neg Ljj$$

$$Mj$$

$$\forall x(Ljx \equiv Lkx)$$

$$Ljj \equiv Lkj$$

$$Mj \supset Lkj$$

| | |
|---|---|
| $\neg Mj$ | $Lkj$ |
| $\#$ | |

| | |
|---|---|
| $Ljj$ | $\neg Lkj$ |
| $Lkj$ | $\neg Ljj$ |
| $\#$ | $\#$ |

(b) (ii)

$\forall x(\neg Mx \equiv x = k) \wedge Bk$

$\neg \forall x((\neg Mx \wedge Bx) \equiv x = k)$

$\exists x \neg((\neg Mx \wedge Bx) \equiv x = k)$

$\neg((\neg Ma \wedge Ba) \equiv a = k)$

$\forall x(\neg Mx \equiv x = k)$

$Bk$

$\neg Ma \equiv a = k$

- $\neg Ma \wedge Ba$
  $\neg a = k$
  $\neg Ma$
  $Ba$
  - $\neg Ma$
    $a = k$
    #
  - $\neg\neg Ma$
    $\neg a = k$
    #
- $\neg(\neg Ma \wedge Ba)$
  $a = k$
  - $\neg\neg Ma$
    $\neg a = k$
    #
  - $\neg Ma$
    $a = k$
    - $\neg\neg Ma$
      #
    - $\neg Ba$
      #

(b) (iii)

$\neg Bk \supset Mj$

$\exists x Lxj \supset \neg Mj$

$\forall x(\exists y(Lyj \wedge By \wedge Lxy) \supset \neg Lkx)$

$\forall x(Lkx \supset x=j)$

$\neg\neg Lkk$

$Lkk \supset k=j$

$\neg Lkk$ #

$k=j$

$\forall x(\exists y(Lyk \wedge By \wedge Lxy) \supset \neg Lkx)$

$\exists y(Lyk \wedge By \wedge Lky) \supset \neg Lkk$

$\neg Lkk$ #

$\neg\exists y(Lyk \wedge By \wedge Lky)$

$\forall y \neg(Lyk \wedge By \wedge Lky)$

$\neg(Lkk \wedge Bk \wedge Lkk)$

$\neg(Lkk)$ #

$\neg(Bk \wedge Lkk)$

$\neg Lkk$ #

$\neg Bk$

$\neg\neg Bk$ #

$Mj$

$\neg Mj$ #

$\neg\exists x Lxj$

$\forall x \neg Lxj$

$\forall x \neg Lxk$

$\neg Lkk$

(b) (iv)

$\forall x(\exists y Lyx \supset \forall y Lxy)$

$\exists x \forall y(\neg \exists z Lzy \equiv y = x)$

$\neg \exists x \forall y(y = x)$

$\forall x \neg \forall y(y = x)$

$\forall y(\neg \exists z Lzy \equiv y = a)$

$\neg \forall y(y = a)$

$\exists y \neg y = a$

$\neg b = a$

$\neg \exists z Lza \equiv a = a$

- $\neg\neg \exists z Lza$
  $\neg a = a$
  #
- $\neg \exists z Lza$
  $a = a$
  $\exists y Lya \supset \forall y Lay$
  $\neg \exists z Lzb \equiv a = b$
  - $\neg \exists z Lzb$
    $a = b$
    #
  - $\neg\neg \exists z Lzb$
    $\neg a = b$
    $\exists z Lzb$
    $\exists y Lyb \supset \forall y Lby$
    - $\neg \exists y Lyb$
      $\forall y \neg Lyb$
      $Lcb$
      $\neg Lcb$
      #
    - $\forall y Lby$
      $Lba$
      $\forall z \neg Lza$
      $\neg Lba$
      #

3.

| | R | S | T | NT |
|---|---|---|---|---|
| x is y's father | N | N | N | N |
| x is a brother of y | N | N | N | N |
| x is y's only sibling | N | Y | N | N |
| x and y have no common ancestor | N | Y | N | N |
| x is an ancestor of y | N | N | Y | N |
| x loves y ≡ y loves x | Y | Y | N | N |
| x loves y ∨ y loves x | N | Y | N | N |
| ∀z (x loves z ≡ z loves y) | N | N | N | N |
| x loves John ≡ y loves Jane | N | N | N | N |
| Majority pref. | N | N | N | N |

4.

(a) This only holds if 5 black socks and 1 white sock so the answer is 0

(b) Pr (2B | 1B) = Pr (2B) / Pr (1B) = 0.25/0.75 = 1/3

(c) Pr (2B | 1BM) = Pr (2B ∧ 1BM) / Pr (1BM) = (13/196) / (27/196) = 13/27

(d) Use the formula:

$$\Pr(A \mid T) = \Pr(T \mid A)\Pr(A) / [\Pr(T \mid A)\Pr(A) + \Pr(T \mid B)\Pr(B)]$$

$$= (1/2 \times 1/2) / [1/2 \times 1/2 + 2/3 \times 1/2] = 3/7$$

(e) use the same formula as in (d): the answer is

$$(0.9 \times 0.2) / (0.9 \times 0.2) + (0.1 \times 0.8) = 9/13$$