

## 1A Logic Model Answers

1.

- (a)
- (i) A truth-function is a function that takes one or many truth-values as inputs and gives a truth-value as an output.
  - (ii) A set of sentences  $P_1, P_2, \dots, P_n$  tautologically entails a sentence  $Q$  if and only if there is no assignment of truth values to the atoms of  $P_1, P_2, \dots, P_n$  and  $Q$  on which  $P_1, P_2, \dots, P_n$  are all true and  $Q$  is false.
  - (iii) The material conditional  $\supset$  is the two-place propositional connective such that  $P \supset Q$  is true unless  $P$  is true and  $Q$  is false.
  - (iv) The metalanguage is the language in which one describes properties and relations of elements of the object language e.g. tautological entailment.
- (b)
- (i) A system that is sound but not complete is the system that allows nothing to be deduced from anything.
  - (ii) A system that is complete but not sound is the system that allows anything to be deduced from anything.

(c) To say that a truth-function can be expressed by  $\wedge$ ,  $\vee$  and  $\neg$  is to say that every truth function is some combination of these truth-functions. To prove it: consider a truth-function  $G$  that takes  $n$  truth-values to a truth-value. Then  $G$ 's truth-table is as follows:

$P_1$	$P_2$	...	$P_n$	$G$
T	T	...	T	$X_1$
F	T	...	T	$X_2$
T	F	...	T	$X_3$
F	F	...	T	$X_4$
...	...	...	...	...
...	...	...	...	...
F	F	...	F	$X_k$

Each of  $X_1, X_2, \dots, X_k$  is either T or F. Consider all the rows  $j_1, j_2, \dots, j_q$  in which  $X_j = T$ . Call each such row a *good row*. Now define for each  $i$ :

$$\begin{aligned}
H(P_i, m) &= P_i \text{ if } P_i \text{ is T in row } m \\
H(P_i, m) &= \neg P_i \text{ if } P_i \text{ is F in row } m \\
E(m) &= H(P_1, m) \wedge H(P_2, m) \wedge \dots \wedge H(P_n, m)
\end{aligned}$$

Then  $G(P_1, \dots, P_n) = E(j_1) \vee E(j_2) \vee \dots \vee E(j_q)$  where  $j_1, j_2, \dots, j_q$  index all and only the good rows. And clearly  $E(j_1) \vee E(j_2) \vee \dots \vee E(j_q)$  involves only  $\wedge, \vee$  and  $\neg$ .

2. (a)

$$(i) \quad Lkj \equiv \neg M$$

$$(ii) \quad \forall x (\neg \exists y Lxy \supset (\neg Lxj \wedge \neg Lxk))$$

$$(iii) \quad \neg Mj \supset \forall x (Lkx \equiv x = j)$$

$$(iv) \quad \exists x \exists y \forall z (((z \neq y \vee z \neq x) \wedge Lzj) \equiv (z = x \vee z = y)) \wedge Mx \equiv \neg My$$

$$(v) \quad \exists x \exists y \forall z ((Bz \equiv z = x) \wedge (Mz \equiv z = y) \wedge (Lzy \supset \neg Lzx))$$

$$(vi) \quad \forall x \forall y ((Bx \wedge By \wedge y \neq x \wedge Lxk \wedge Lyk) \supset (Lxy \wedge Lyx))$$

(b) (i)

$$M_j \wedge \forall x (L_{jx} \equiv L_{kx})$$

$$\forall x (M_x \supset L_{kx})$$

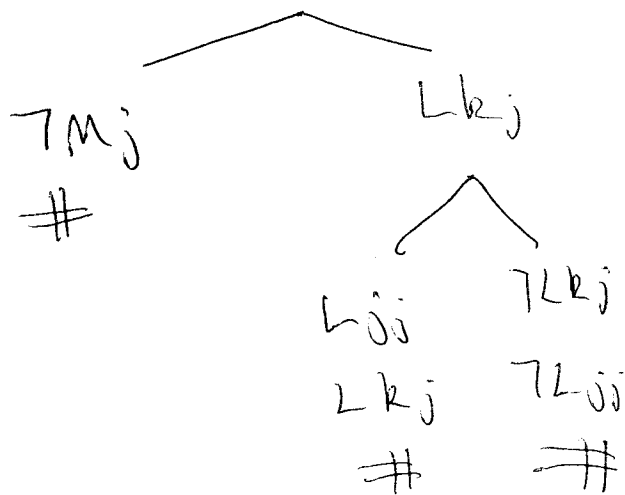
$$\neg L_{jj}$$

$$M_j$$

$$\forall x (L_{jx} \equiv L_{kx})$$

$$L_{jj} \equiv L_{kj}$$

$$M_j \supset L_{kj}$$



(b) (ii)

$$\forall x (\neg Mx \equiv x=k) \wedge Bk$$

$$\neg \forall x ((\neg Mx \wedge Bx) \equiv x=k)$$

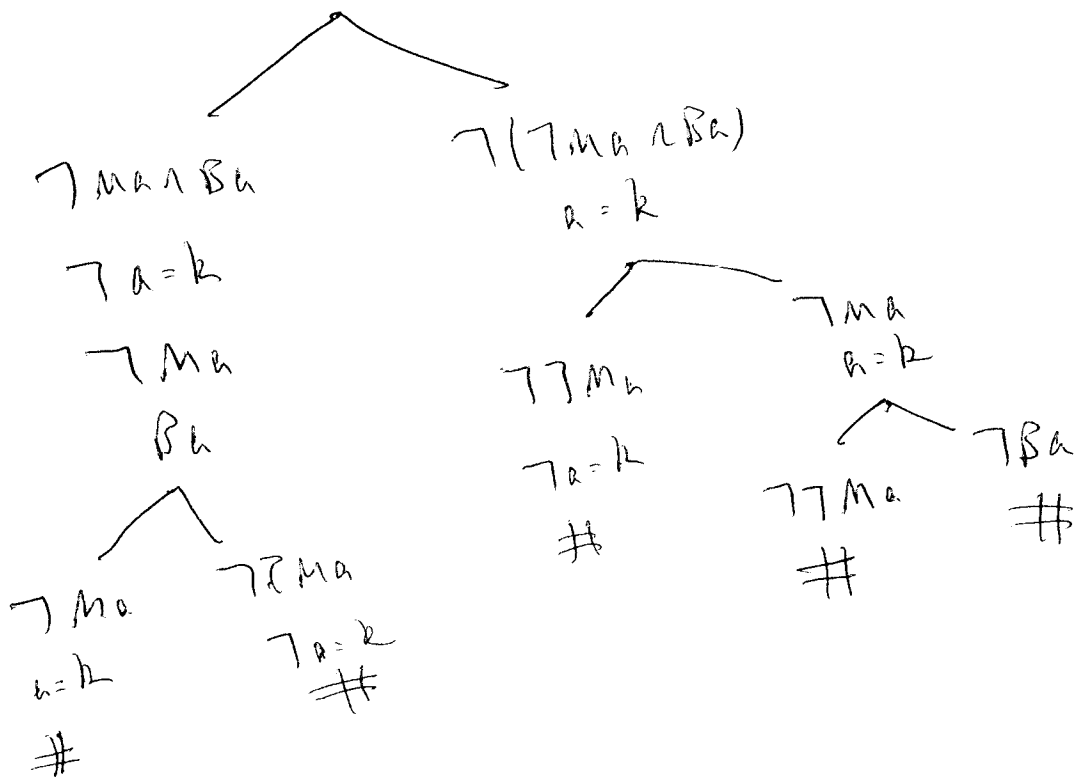
$$\exists x \neg ((\neg Mx \wedge Bx) \equiv x=k)$$

$$\neg ((\neg Ma \wedge Ba) \equiv a=k)$$

$$\forall x (\neg Mx \equiv x=k)$$

$$Bk$$

$$\neg Ma \equiv a=k$$



(b) (iii)

$$\neg Bk \supset Mj$$

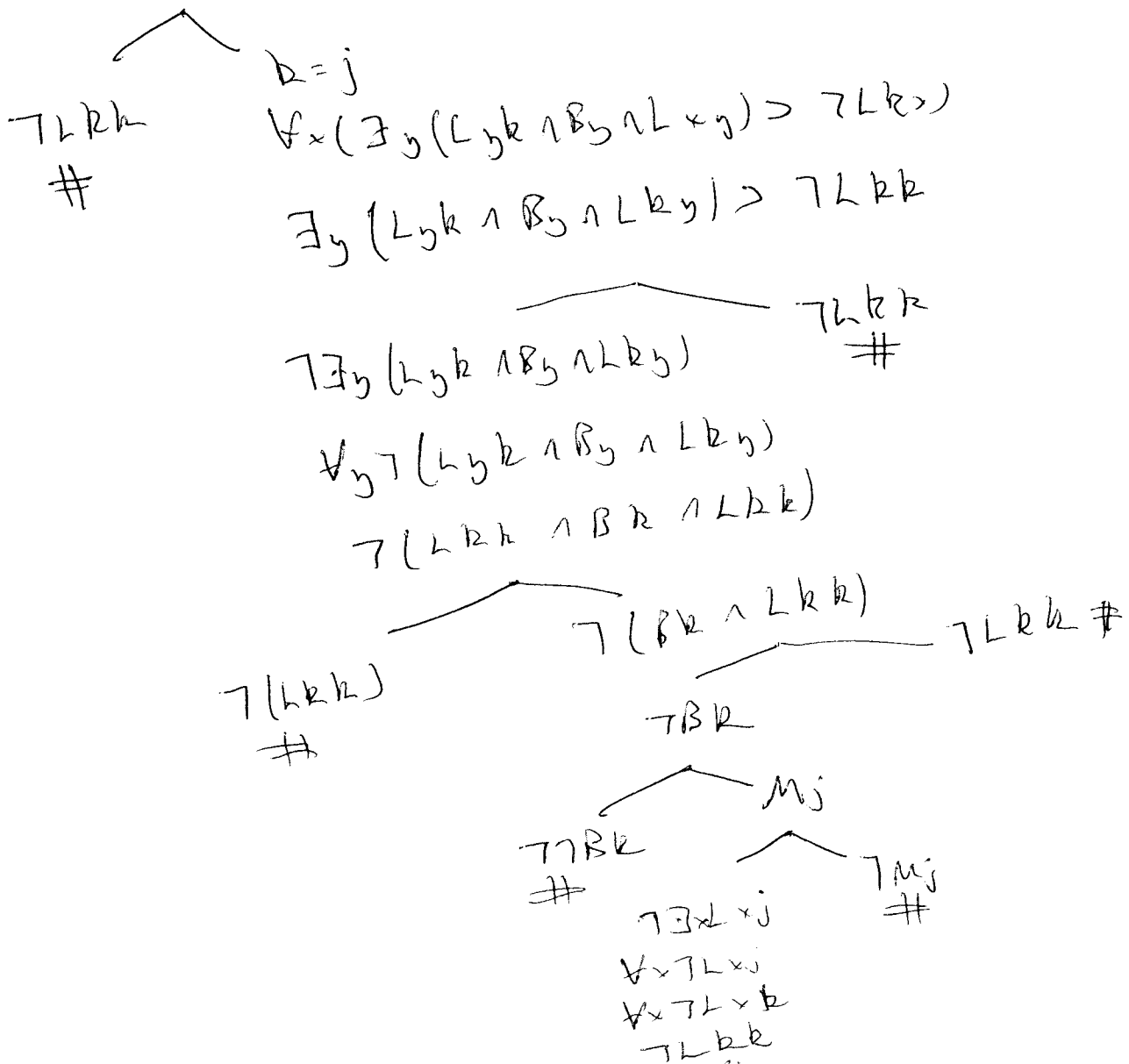
$$\exists x Lxj \supset \neg Mj$$

$$\forall x (\exists y (Lyj \wedge Bx \wedge Lxy) \supset \neg Lkx)$$

$$\forall x (Lkx \supset x=j)$$

$$\neg \neg Lkk$$

$$Lkk \supset k=j$$



(b) (iv)

$$\forall x (\exists y L y x \supset \forall y L x y)$$

$$\exists x \forall y (\neg \exists z L z y \equiv y = x)$$

$$\neg \exists x \forall y (y = x)$$

$$\forall x \neg \forall y (y = x)$$

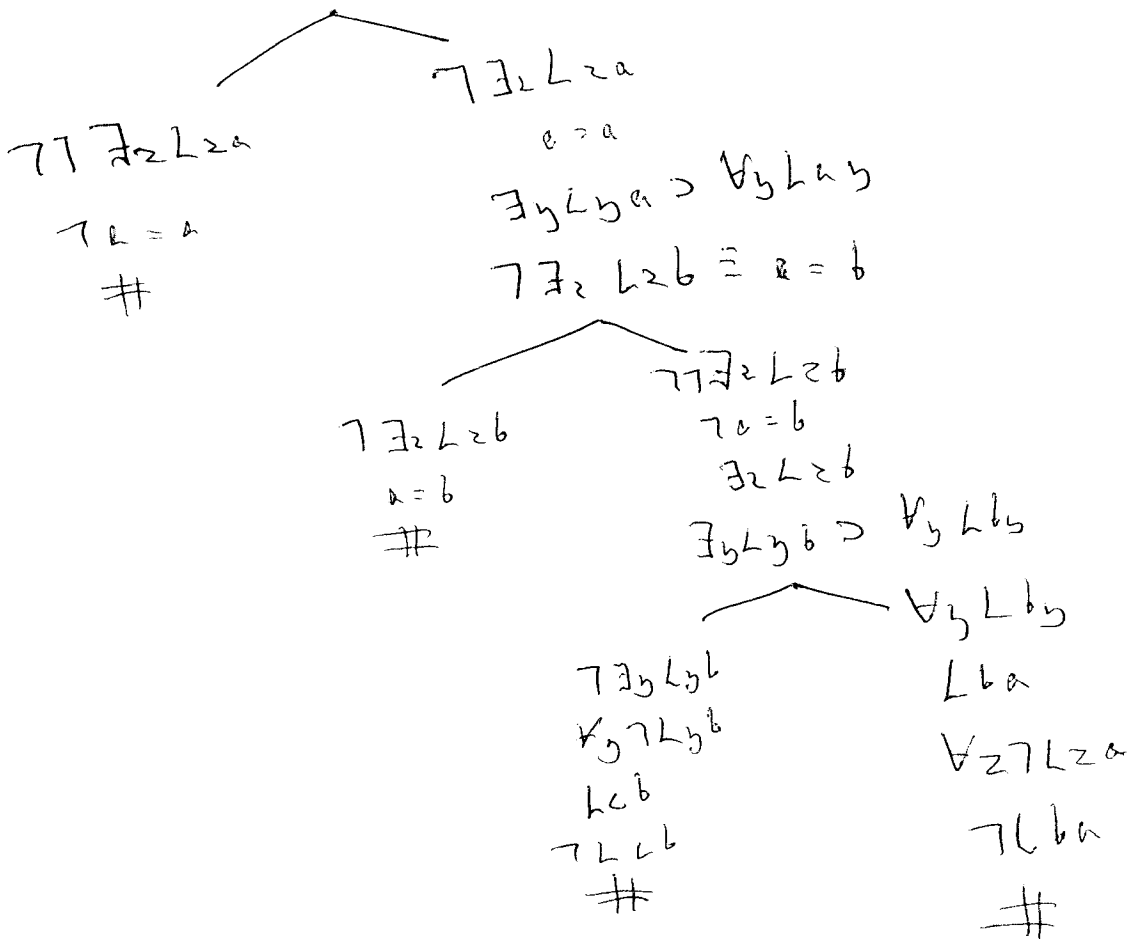
$$\forall y (\neg \exists z L z y \equiv y = a)$$

$$\neg \forall y (y = a)$$

$$\exists y \neg y = a$$

$$\neg b = a$$

$$\neg \exists z L z a \equiv a = a$$



3.

	R	S	T	NT
x is y's father	N	N	N	N
x is a brother of y	N	N	N	N
x is y's only sibling	N	Y	N	N
x and y have no common ancestor	N	Y	N	N
x is an ancestor of y	N	N	Y	N
x loves y $\equiv$ y loves x	Y	Y	N	N
x loves y $\vee$ y loves x	N	Y	N	N
$\forall z$ (x loves z $\equiv$ z loves y)	N	N	N	N
x loves John $\equiv$ y loves Jane	N	N	N	N
Majority pref.	N	N	N	N

4.

(a) This only holds if 5 black socks and 1 white sock so the answer is 0

$$(b) \Pr(2B | 1B) = \Pr(2B) / \Pr(1B) = 0.25/0.75 = 1/3$$

$$(c) \Pr(2B | 1BM) = \Pr(2B \wedge 1BM) / \Pr(1BM) = (13/196) / (27/196) = 13/27$$

(d) Use the formula:

$$\Pr(A | T) = \Pr(T | A) \Pr(A) / [\Pr(T | A) \Pr(A) + \Pr(T | B) \Pr(B)]$$

$$= (1/2 \times 1/2) / [1/2 \times 1/2 + 2/3 \times 1/2] = 3/7$$

(e) use the same formula as in (d): the answer is

$$(0.9 \times 0.2) / (0.9 \times 0.2) + (0.1 \times 0.8) = 9/13$$