

PHILOSOPHY TRIPOS Part IA

Tuesday 27 May 2003

9 to 12

Paper 3

LOGIC

*Answer **four** questions only; at least **one** from each section.*

*Write the number of the question at the beginning of each answer.
Please answer **all** parts of each numbered question chosen.*

**You may not start to read the questions
printed on the subsequent pages of this
question paper until instructed that you
may do so by the Invigilator**

SECTION A

- 1 Define the following notions:
- (a) truth-functional connective;
 - (b) tautology;
 - (c) tautological entailment;
 - (d) expressive adequacy (of a set of connectives).
Show that the set of connectives $\{ \neg, \wedge \}$ is expressively adequate.
- 2
- (a) Define transitivity, symmetry, reflexivity. Give two examples of an equivalence relation.
 - (b) Suppose that R is an equivalence relation. Show that $\forall x \forall y \forall z ((Rxy \wedge Rxz) \rightarrow Ryz)$.
 - (c) Let us say that a relation R is complete if $\forall x \forall y (Rxy \rightarrow Ryx)$. Taking your domain to be the set of people, either give examples of relations which have the following properties or explain why no examples are possible:
 - (i) complete and symmetric;
 - (ii) complete and irreflexive;
 - (iii) transitive, not symmetric, and not complete;
 - (iv) transitive and symmetric but not complete;
 - (v) complete and symmetric but not an equivalence relation.
- 3 Translate the following sentences into $QL^=$ (the language of the predicate calculus with identity). Be sure to state the translation scheme that you use.
- (a) Only if Bertrand is a philosopher are all logicians philosophers.
 - (b) No philosophers who are not logicians are admired by Bertrand.
 - (c) Every philosopher admires some logician.
 - (d) Some philosophers are admired by no logician.
 - (e) Any philosopher who admires some logician admires Frank.
 - (f) Some logician other than Bertrand admires someone other than himself.
 - (g) Only Bertrand admires all those logicians who do not admire themselves.
 - (h) Whoever admires all logicians only admires Frank and Bertrand.
 - (i) No one other than Frank and Bertrand is admired by any philosopher who admires some logician.
 - (j) The philosopher who admires Frank is a logician.
 - (k) Frank is not the logician who admires Bertrand.
 - (l) Only if Bertrand and Frank are different people are there exactly two logicians who are philosophers.
- 4 Show the following arguments are valid by translating them into $QL^=$ (the language of the predicate calculus with identity) and using predicate trees.
- (a) All cricketers have good hand-eye co-ordination. No one clumsy has good hand-eye co-ordination. So no cricketer is clumsy.
 - (b) Any true philosopher admires some logician. Some students admire only existentialists. No existentialists are logicians. Hence not all students are true philosophers.

[TURN OVER for continuation of question 4]

- (c) Some hockey players admire anyone who has played for England. Fred has no admirers. So Fred has not played for England.
- (d) Angharad and Bethan, and they alone, love Caradoc. Someone who loves Caradoc kissed him. So either Angharad or Bethan kissed Caradoc.
- (e) All logicians are philosophers; hence any logician's daughter is a philosopher's daughter.
- (f) If the King of France exists, he is a bald man. Bald men are sexy. Hence the King of France is sexy, if he exists.
- 5 (a) Define conditional probability. Define what is meant in probability theory by saying that two events are 'independent'.
- (b) Prove that $P(A) = P(B) P(A|B) + P(\neg B) P(A|\neg B)$.
- (c) I choose three different letters at random from the alphabet. What is the probability that I have chosen letters in alphabetical order? (For example, I have chosen letters in alphabetical order if I choose A then D then X. I have *not* chosen letters in alphabetical order if I choose C then B then A.) Explain your answer.

SECTION B

- 6 Does Russell give a correct account of the meaning of sentences such as 'The present King of France is bald'?
- 7 Must any meaningful sentence be verifiable?
- 8 Explain why the material conditional is the only truth-functional connective which is a possible candidate for representing the indicative conditional of ordinary language. How good a candidate is it?
- 9 Does it matter whether analyticity is a coherent notion?
- 10 Explain, with examples, the distinctions between the following: sentence token, sentence type, proposition, statement.

END OF PAPER