PHILOSOPHY TRIPOS Part IA

Friday 31 May 2019 13.30 – 15:30

Paper 5

FORMAL METHODS

*Answer **all** questions in Section A. Each question in Section A is worth 9 marks.*

*Answer **two** questions from Section B. Each question in Section B is worth 20 marks.*

Write the number of the question at the beginning of each answer.

STATIONERY REQUIREMENTS
20 page answer book × 1
Rough work pad

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

Answer all questions in section A.

(1) Which of the following claims are true, and which are false? Briefly explain your answers:
   (a) Every invalid argument can be turned into a valid argument by adding a premise.
   (b) Some valid argument can be turned into an invalid argument by adding a premise.
   (c) Every sound argument can be turned into an unsound argument by adding a premise.

(2) Consider an interpretation, whose domain is just the numbers 1, 2, and 3, and where the predicate 'R' is to be true of, and only of:

$$\langle 1,1\rangle, \langle 1,2\rangle, \langle 1,3\rangle, \langle 2,3\rangle$$

Given this interpretation, state the truth values of each of these sentences (you do not need to explain your answers):
   (a) $\exists x Rxx$
   (b) $\exists x \forall y Rxy$
   (c) $\forall x \exists y Rxy$
   (d) $\forall x \exists y Ryx$
   (e) $\exists x \exists y (Rxy \wedge \neg Ryx)$

(3) Construct complete truth tables for each of these three sentences:

$$\neg(A \rightarrow B), \neg A \rightarrow B, A \rightarrow \neg B$$

Use these truth tables to determine whether each of the following claims is true or false:
   (a) $\neg(A \rightarrow B) \vDash \neg A \rightarrow B$
   (b) $\neg(A \rightarrow B) \vDash A \rightarrow \neg B$
   (c) $\neg A \rightarrow B \vDash \neg(A \rightarrow B)$
   (d) $\neg A \rightarrow B \vDash A \rightarrow \neg B$
   (e) $A \rightarrow \neg B \vDash \neg(A \rightarrow B)$
   (f) $A \rightarrow \neg B \vDash \neg A \rightarrow B$

(4) For each of the following relations on the domain of countries, say whether they are reflexive, whether they are symmetric, and whether they are transitive:
   (a) x has exactly the same area as y
   (b) x has at least as many inhabitants as y
   (c) x has a land border with y

(5) Two cards are drawn at random, without replacement, from a standard deck of cards (i.e. 4 of the 52 cards are aces). What is the probability that:
   (a) both are aces?
   (b) exactly one is an ace?
   (c) at least one is an ace?

SECTION B

Answer any **two** questions from section B.

(6) Attempt both parts of this question. Throughout, use this symbolisation key:

domain: all physical objects

C: $\_\_\_\__1$ is a cup

S: $\_\_\_\__1$ is a saucer

A: $\_\_\_\__1$ belongs to Anna

O: $\_\_\_\__1$ is on top of $\_\_\_\__2$

(a) Offer natural-language renditions of the following sentences of FOL. Make the natural-language sentences as natural as possible, whilst keeping the same truth-conditions as the FOL sentences:

(i) $\forall x(Ax \to \neg(Cx \lor Sx))$

(ii) $\forall x((Cx \land \exists y(Sy \land Oxy)) \to Ax)$

(iii) $\exists x(Cx \land Ax \land \forall y((Cy \to Ay) \to x = y))$

(iv) $\exists x(Cx \land \forall y(Cy \to x = y) \land Ax)$

(v) $\exists x(\forall y(Sx \leftrightarrow x = y) \land \forall y((Cy \land Ay) \to \neg Oyx))$

(b) Symbolise all of the following English sentences as best you can in FOL. Comment on any difficulties you encounter:

(i) Anna owns a cup but no saucer

(ii) All of Anna's cups are on saucers

(iii) Anna's cup is on a saucer

(iv) Anna's saucer has exactly two things on it

(v) The cup on Anna's saucer indeed belongs to Anna

(7) Using the formal proof system from *forallx:Cambridge*, show each of the following:

(a) $\exists xFx \to \forall yGy \vdash \forall x\forall y(Fx \to Gy)$

(b) $\neg\exists x(Fx \land Gx) \vdash \forall x(Gx \to \neg Fx)$

(c) $\forall x\exists yRxy, \forall x\neg Rxx \vdash \exists x\exists y\neg x = y$

(d) $Fb, Lab, \exists x\forall y(Lay \to x = y) \vdash \forall x(Lax \to Fx)$

(8) Answer both parts of this question.

(a) Jelly beans are manufactured in two colours: red and green. They are manufactured so that 5% of the beans of one colour are bitter, and 10% of the beans of the other colour are bitter; but you don't know which is which. What probability should you assign to the hypothesis that 10% of red beans are bitter, in each of the following distinct scenarios:

(i) You eat a single red bean, and it is bitter.

(ii) You eat a single red bean, and it is not bitter.

(iii) You eat two red beans, and neither is bitter.

(b) A D20 is a fair, twenty-sided die, whose faces are numbered 1 through 20.

(i) You roll two D20. What is the probability that they both show the same number?

(ii) You roll eight D20. What is the probability that at least two of the eight dice show the same number?

(9) Answer all parts of this question.

(a) Let L be the set of all labradors. Let B be the set of all bulldogs. (Note that no labrador is a bulldog, and no bulldog is a labrador.) Using the following symbolisation key:

T: $\underline{\quad}_1$ has a longer tail than $\underline{\quad}_2$

f: Fido (who is a labrador)

write down set-theoretic expressions for the following sets:

(i) The set of all bulldogs with tails longer than Fido's.

(ii) The set of all bulldogs with tails longer than every labrador's.

(iii) The set of ordered pairs, one of which is a labrador and the other of which is a bulldog.

(iv) The set of ordered pairs, one of which is a labrador other than Fido and the other of which is a bulldog.

(b) Let $A = \{0, 2, 4, 6\}$, $B = \{0, 1, 2, \{4\}, \{6\}\}$, $C = \{\{0\}\}$. Calculate the members of the following sets:

(i) $(A \cup B) \cap C$

(ii) $B \times C$

(iii) $\wp(A) - \wp(B)$

(iv) $\wp(C) \times \wp(B - A)$

(c) What are the members of $\wp(\wp(\varnothing))$?

(d) Show that, if $X \subseteq Y$ and $Y \cap Z = \varnothing$, then $X \cap Z = \varnothing$

END OF PAPER