PHILOSOPHY TRIPOS Part IA

TUESDAY 29 May 2018 09.00 – 12.00

Paper 3

LOGIC

Answer **three** *questions only; at least* **one** *from each of* **sections A and B***.*

Write the number of the question at the beginning of each answer.

Each question has equal weight. For Section A (formal questions) the number in square brackets after each component of a question indicates the relative weight of that component.

STATIONERY REQUIREMENTS
20 Page Answer Book x 1
Rough Work Pad

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1. Attempt all parts of this question.

   (a) Using the following symbolisation key,

   a: Archie
   b: Bertie
   c: Cookie
   Dx: $\_\_\_\__x$ is a dog
   Px: $\_\_\_\__x$ is a pug
   Txy: $\_\_\_\__x$ is a tail of $\_\_\_\__y$
   Lxy: $\_\_\_\__x$ loves $\_\_\_\__y$

   symbolise each sentence of each of the following arguments. Comment on any difficulties you encounter.

   (i) Every pug is a dog. So every pug's tail is a dog's tail.
   (ii) No pug has a tail. After all, a pug with a tail would have to be a dog. But no dog has a tail.
   (iii) Archie the pug loves himself and himself alone. Nothing else loves any pug. So exactly one pug is loved.
   (iv) If Archie loves at all, he loves exactly one thing. So if Archie loves Cookie—who, it must be noted, is not a pug—then Archie loves no pugs.
   (v) Bertie loves Cookie. Cookie loves Bertie. Bertie is a pug. Archie loves nothing that loves anything that loves a pug. So Archie does not love Bertie. [40]

   (b) Using the formal proof system from *forallx*, provide formal proofs to vindicate each of the arguments from part (a). You may use both basic and derived rules. [45]

2. Attempt all parts of this question.

   (a) Using the following symbolisation key,

   Rxy: $\_\_\_\__x$ can see $\_\_\_\__y$

   symbolise each sentence of each of the following arguments. Comment on any difficulties you encounter.

   (i) Everything sees something. So everything is seen by something.

TURN OVER

(ii) Nothing sees everything. So if there is something that can see nothing, then it cannot be seen.

(iii) Whatever sees anything at all sees everything that sees it. So everything sees itself.

(iv) Everything sees exactly one thing. So if there are exactly two things, then exactly two things are seen.

(v) Everything sees more than one thing. Everything is seen by more than one thing. So there are at least three things.

(vi) Nothing sees itself. Everything sees exactly one thing. Exactly one thing is unseen; everything else is seen by exactly one thing. So at least two things can see each other. [30]

(b) Provide counter-interpretations to show that each of the arguments from part (a) is invalid. [30]

(c) Using the same symbolisation key as in part (a), symbolise the following sentences:

(i) Some unseen thing sees everything.

(ii) Something sees all and only those things which do not see themselves. [10]

(d) Provide informal arguments to show that no interpretation makes either (i) or (ii) of part (c) true. [15]

3. Attempt all parts of this question.

(a) Which of the following statements are true? (Answer just by writing 'True' or 'False' for each statement.)

(i) $\varnothing \in \varnothing$

(ii) $\varnothing \subseteq \varnothing$

(iii) $\varnothing \subseteq \{\varnothing\}$

(iv) $\varnothing \in \{\{\varnothing\}\}$

(v) $\varnothing \in \varnothing \cap \{\varnothing\}$

(vi) $\varnothing \in \varnothing \cup \{\varnothing\}$

(vii) $\varnothing = \varnothing \times \{\varnothing\}$

(viii) $\{\varnothing\} \in \{\varnothing\} \cap \{\{\varnothing\}\}$

(ix) $\{\varnothing\} \subseteq \{\varnothing\} \cup \{\{\varnothing\}\}$

(x) $\{\varnothing\} = \{\varnothing\} \times \{\{\varnothing\}\}$ [20]

(b) Let $A = \{1, 2, 3\}$, $B = \{2, 3\}$, $C = \{3, 4, 5\}$, and $D = \{6\}$. Calculate the members of each of the following sets:

TURN OVER

(i) $(A \cap B) \cup (C \cap D)$
(ii) $A \times B$
(iii) $\wp(B \cap C)$
(iv) $\wp(\wp(C \cap D))$
(v) $(A - C) \times D$ [20]

(c) Let R be a binary relation defined on a set X. Define what it means to say that:

(i) R is reflexive.
(ii) R is symmetric.
(iii) R is transitive.
(iv) R is an equivalence relation. [10]

(d) Say that a binary relation R on X is *serial* iff for any $x \in X$ there exists some $y \in X$ such that xRy. Say that R is *circular* iff, for any $x, y, z \in X$, if xRy and yRz then zRx. Use these definitions to prove the following, for any binary relation R.

(i) If R is circular and symmetric, then R is transitive.
(ii) If R is circular, serial and symmetric, then R is reflexive.
(iii) R is reflexive and circular iff R is an equivalence relation. [35]

4. Attempt all parts of this question. In parts (b)–(d), explain your answers, and state any additional assumptions you have made.

(a) Define conditional probability and state Bayes' Theorem. [10]

(b) Two fair six-sided dice are rolled repeatedly. The sums of the two numbers appearing on the dice are observed (for example, if one die shows 2 and the other shows 5, then the sum is 7).

(i) What is the probability that a sum of 7 is observed on the first roll?
(ii) When rolled repeatedly, what is the probability that a sum of 7 is observed before any sum of 8 has been observed? [20]

(c) Malika lives in a population where 1 in $10,000$ people has a particular condition, C. Malika is given a test to determine whether she has C. The test is 90% accurate, in this sense: if someone has C, there is a probability of 0.9 that the test will give a positive report; whereas, if someone does not have C, there is a probability of 0.1 that the test will report positively. Malika receives a positive report. What is the probability that she has C? [20]

(d) A box contains three coins with a head on each side, four coins with a tail on each side, and two fair coins. One of these nine coins is selected at random, and tossed repeatedly. Calculate the probability that:

(i) the selected coin will land heads on its first toss.

(ii) the selected coin will land heads on its second toss, given that it landed heads on its first toss.

(iii) the selected coin will land heads on its third toss, given that it landed heads on both of its first two tosses. [35]

SECTION B

5. Should we adopt a substitutional view of quantifiers?

6. 'Given any necessary a posteriori truth, we can always know a priori *that* it is either necessarily true or necessarily false; it is just that we can only know a posteriori *which* it is. So Kripke's examples of necessary a posteriori truths do not undermine the close connection between necessity and a priority.' Discuss.

7. How much, if anything, is lost by treating indicative conditionals as material conditionals?

8. EITHER (a) 'The sentence "the largest prime number does not exist" is true. Therefore Russell's theory of descriptions is false.' Discuss.
OR (b) Given that there is no King of France, is it coherent to maintain that 'I spent yesterday with the present King of France' is false, whereas 'The present King of France is bald' is neither true nor false?

END OF PAPER