

PHILOSOPHY TRIPOS Part II

Tuesday 7 June 2005

9 to 12

Paper 7

MATHEMATICAL LOGIC

Answer **three** questions only.

Write the number of the question at the beginning of each answer. If you are answering an either/or question, indicate the letter as well.

STATIONERY REQUIREMENTS

20 Page Answer Book x 1

Rough Work Pad

**You may not start to read the questions
printed on the subsequent pages of this
question paper until instructed that you
may do so by the Invigilator**

- 1 Should the notions of proof and deduction be decidable? Should a logic be axiomatisable? Should a logic be decidable?
- 2 Using examples, explain the notions of *non-standard model* and *categorical theory*. Describe a non-standard model for first-order Peano Arithmetic. Is its existence important?
- 3 **Either** (a) Is the notion of computability vague? Explain the significance of your answer.
Or (b) Outline a proof that every recursive function is Turing-computable.
- 4 **Either** (a) Do Gödel's incompleteness theorems show that meaning isn't use?
Or (b) Does Gödel's first incompleteness theorem show that the mind is not a finite machine?
- 5 Explain and illustrate the relevance of the following ideas to Gödel's first incompleteness theorem:
 - (a) a beta-function;
 - (b) omega-consistency;
 - (c) omega-incompleteness.
- 6 What reasons are there to believe the set-theoretic axioms of extensionality, infinity and choice? Does it matter if there are different reasons for different axioms?
- 7 'Arithmetic is more certain than set theory. So reducing arithmetic to set theory is pointless.' Discuss.
- 8 Is Hilbert's programme completely undermined by Gödel's incompleteness theorems?
- 9 What does Gentzen's proof of the consistency of arithmetic tell us?
- 10 Is there a problem in using second-order quantifiers which are interpreted as quantifying over all arbitrary subsets of infinite domains?

END OF PAPER