

PHILOSOPHY TRIPOS Part II

Tuesday 28 May 2002

9 to 12

Paper 7

MATHEMATICAL LOGIC

Answer **three** questions only.

Write the number of the question at the beginning of each answer. If you are answering an either/or question, indicate the letter as well.

**You may not start to read the questions
printed on the subsequent pages of this
question paper until instructed that you
may do so by the Invigilator**

- 1 Sketch a proof of the strong completeness of some deductive system for first-order logic without identity.
- 2 Is second-order logic really set theory?
- 3 Should a logic be decidable? Should a logic be compact?
- 4 What does it mean to say that a formal theory is categorical? Are first-order Peano Arithmetic and second-order Peano Arithmetic categorical? Explain your answers and comment on their philosophical significance.
- 5 What does Gentzen's proof of the consistency of arithmetic tell us?
- 6 **Either** (a) Sketch a proof of the cut elimination theorem for first-order predicate calculus.
Or (b) How do systems of logic with and without the cut rule differ?
- 7 In what sense, if any, is it possible to reduce mathematics to set theory?
- 8 'The iterative conception of set is entirely natural, consistent, and free from paradox.' Discuss.
- 9 **Either** (a) Construct an unprovable sentence of first-order Peano Arithmetic. Show it is unprovable.
Or (b) What do Gödel's incompleteness theorems show us about the mind?
- 10 Are the computable functions exactly (a) the primitive recursive functions, or (b) the recursive functions, or (c) the partial recursive functions, or (d) none of these?

END OF PAPER