PHT2/7

PHILOSOPHY TRIPOS Part II

Wednesday 28th May 2014   09.00 – 12.00

Paper 7

MATHEMATICAL LOGIC

Answer **three** *questions only.*

Write the number of the question at the beginning of each answer. If you are answering the either/or question, indicate the letter as well.

STATIONERY REQUIREMENTS
20 Page Answer book x 1
Rough Work Pad

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1. Answer all parts of the question:

   (a) What is it for a theory to be (i) axiomatizable, (ii) categorical, (iii) decidable?

   (b) Outline the following three theories: first-order Peano arithmetic, first-order complete arithmetic, second-order Peano arithmetic. Which of these theories is (i) axiomatizable, (ii) categorical, (iii) decidable? Give brief reasons.

   (c) Can a theory be categorical but not complete? Explain your answer.

2. What is meant by a maximally consistent, ω-complete set of sentences? Explain the role of such sets in a proof of the completeness of a deductive system for first-order logic without identity.

3. ‘Second-order logic is not wholly general, since it is impossible for a second-order theory to be about all sets. Hence second-order logic is not logic.’ Discuss.

4. Outline an account of how arithmetic and analysis can be embedded in set theory.

5. Sketch an account of cardinal arithmetic, including definitions of the operations of addition, multiplication and exponentiation. State and prove Cantor’s theorem.

6. ‘We cannot justify the axiom scheme of replacement by appeal to the iterative conception of set; but the iterative conception of set is the best conception of set we have; so we should refrain from using the axiom scheme of replacement.’ Discuss.

7. Robinson Arithmetic **Q** is a first-order theory in the language of basic arithmetic with the following axioms:

$$\forall x(\bar{0} \neq \mathrm{s}x)$$
$$\forall x\forall y(\mathrm{s}x = \mathrm{s}y \rightarrow x = y)$$
$$\forall x(x \neq \bar{0} \rightarrow \exists y\ (x = \mathrm{s}y))$$
$$\forall x(x + \bar{0} = x)$$
$$\forall x\forall y(x + \mathrm{s}y = \mathrm{s}(x + y))$$
$$\forall x(x \times \bar{0} = \bar{0})$$
$$\forall x\forall y(x \times \mathrm{s}y = (x \times y) + x)$$

   Prove that for any closed atomic sentence φ in the language of the theory either **Q** ⊢ φ or **Q** ⊢ ¬φ.

8. Show that if a first-order theory T is (primitive recursively) axiomatizable, contains the language of basic arithmetic, is sound, and can capture all primitive recursive functions, then there is a closed sentence G_T such that $T \nvdash G_T$ and $T \nvdash \neg G_T$.

9. ‘Computability is a modal notion; recursiveness is not. So Church’s thesis is false.’ Discuss.

10. EITHER (a) ‘The moral of Skolem’s paradox is that countability is a relative notion.’ Discuss.

    OR (b) Is there any sense in which Gödel’s incompleteness theorems show that the concept *natural number* is indeterminate?

END OF PAPER