

**PHILOSOPHY TRIPOS Part II**

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Thursday 29 May 2003

9 to 12

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Paper 7

MATHEMATICAL LOGIC

Answer **three** questions only.

*Write the number of the question at the beginning of each answer. If you are answering an either/or question, indicate the letter as well.*

**You may not start to read the questions  
printed on the subsequent pages of this  
question paper until instructed that you  
may do so by the Invigilator**

- 1 Outline a proof of the completeness of first-order logic.
- 2 Does the Löwenheim-Skolem theorem show that countability is a relative notion? If so, relative to what?
- 3 Second-order logic cannot be given a complete set of rules. Is this a reason to think it is not really logic?
- 4 What advantages, if any, do second-order theories have over their first-order counterparts?
- 5 What is the relation between the iterative conception of set and the ZFC axioms?
- 6 Sketch a proof that Peano Arithmetic can be embedded in set theory. What is the philosophical significance of this result?
- 7 'Peano arithmetic is obviously consistent since it is true of the natural numbers. So Gentzen's proof that it is consistent is pointless.' Discuss.
- 8 What is Church's thesis? What kind of claim is it?
- 9 **Either** (a) Show that the halting problem for Turing machines is unsolvable. Comment on the significance of this result.  
**Or** (b) How did Gödel construct an arithmetic sentence that in effect says 'I am unprovable'? Outline his proof that such a sentence is unprovable and hence true.
- 10 **Either** (a) Does Gödel's first incompleteness theorem show that minds are not machines?  
**Or** (b) Does Gödel's second incompleteness theorem undermine Hilbert's programme?

END OF PAPER