Answer three questions only.

Write the number of the question at the beginning of each answer. If you are answering an either/or question, indicate the letter as well.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.
What is wrong with the iterative concept of set?

Sketch the main ideas of the arithmetic of infinite cardinal numbers. State and prove Cantor's theorem. Explain what is meant by saying that your proof is impredicative.

Is second-order logic logic?

Either (a) Does Gödel's first incompleteness theorem show that minds cannot be machines?

Or (b) Give a careful statement of Gödel's second incompleteness theorem. Does it refute Hilbert's programme?

Either (a) Outline some way of proving Gödel's first incompleteness theorem that is not a version of Gödel's original proof. Comment briefly on why they are interestingly different.

Or (b) Explain carefully how the notions of (i) a beta-function and (ii) omega-consistency feature in a Gödel-style proof of the first incompleteness theorem.

'Peano arithmetic suffices to prove all the truths that can be established by purely arithmetical reasoning.' Discuss.

Outline a proof that all Turing computable functions are recursive.

'Church's thesis is not a theorem, so there is no point in looking for a proof of it.' Discuss.

Discuss the significance of non-standard models for first-order arithmetic.

Can we quantify over everything?