

**PHILOSOPHY TRIPOS Part II**

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Friday 23 May 2008

09.00 to 12.00

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Paper 7

**MATHEMATICAL LOGIC**

Answer **three** questions only.

*Write the number of the question at the beginning of each answer. If you are answering an either/or question, indicate the letter as well.*

**STATIONERY REQUIREMENTS**

*20 Page Answer Book x 1*

*Rough Work Pad*

**You may not start to read the questions  
printed on the subsequent pages of this  
question paper until instructed that you  
may do so by the Invigilator**

- 1 What is wrong with the iterative concept of set?
- 2 Sketch the main ideas of the arithmetic of infinite cardinal numbers. State and prove Cantor's theorem. Explain what is meant by saying that your proof is impredicative.
- 3 Is second-order logic logic?
- 4 **Either** (a) Does Gödel's first incompleteness theorem show that minds cannot be machines?  
**Or** (b) Give a careful statement of Gödel's second incompleteness theorem. Does it refute Hilbert's programme?
- 5 **Either** (a) Outline some way of proving Gödel's first incompleteness theorem that is not a version of Gödel's original proof. Comment briefly on why they are interestingly different.  
**Or** (b) Explain carefully how the notions of (i) a beta-function and (ii) omega-consistency feature in a Gödel-style proof of the first incompleteness theorem.
- 6 'Peano arithmetic suffices to prove all the truths that can be established by purely arithmetical reasoning.' Discuss.
- 7 Outline a proof that all Turing computable functions are recursive.
- 8 'Church's thesis is not a theorem, so there is no point in looking for a proof of it.' Discuss.
- 9 Discuss the significance of non-standard models for first-order arithmetic.
- 10 Can we quantify over everything?

END OF PAPER