

PHILOSOPHY TRIPOS Part II

Wednesday 27 May 2009

09.00 to 12.00

Paper 7

MATHEMATICAL LOGIC

Answer **three** questions only.

Write the number of the question at the beginning of each answer. If you are answering an either/or question, indicate the letter as well.

STATIONERY REQUIREMENTS

20 Page Answer Book x 1

Rough Work Pad

**You may not start to read the questions
printed on the subsequent pages of this
question paper until instructed that you
may do so by the Invigilator**

- 1 Explain what it is for a logic to be (a) compact and (b) strongly axiomatizable. Can you give an example of a logic which is (i) both compact and strongly axiomatizable, (ii) neither, (iii) strongly axiomatizable but not compact? Justify your answers.
- 2 Compare and contrast first-order and second-order Peano Arithmetic. Which is the better theory?
- 3 **Either** (a) Sketch the leading ideas of a Henkin-style proof of the strong completeness for a deductive system for first-order logic without identity. Show how the Löwenheim-Skolem theorem drops out as a corollary.
Or (b) Is Skolem's Paradox a paradox?
- 4 'There are at least two thoughts behind set theory' (BOOLOS). Discuss.
- 5 **Either** (a) Is the Axiom of Choice true?
Or (b) Is the Axiom of Replacement true?
- 6 **Either** (a) Sketch an account of the arithmetic of ordinal numbers.
Or (b) Sketch an account of the arithmetic of cardinal numbers.
- 7 **Either** (a) Answer all of (i) to (iv):
 - (i) Define the primitive recursive functions and the recursive total functions.
 - (ii) Show that such functions are computable.
 - (iii) Show that there are computable total functions which aren't primitive recursive, and explain why a similar argument can't be used to show that there are computable total functions which aren't recursive.
 - (iv) Are the recursive total functions all the computable total functions?**Or** (b) 'Our only reason for believing Church's Thesis is quasi-empirical: it's just that every attempt to define the computable functions arrives at the same class of functions.' Discuss.
- 8 Outline a proof that first-order Peano Arithmetic is incomplete.
- 9 'The unsolvability of the Halting Problem shows that first-order logic is undecidable.' Explain.
- 10 What was the goal of Hilbert's Programme? Do Gödel's incompleteness theorems show it to be impossible to achieve?

END OF PAPER