RUBRIC

Time allowed: 2 hours

Answer **all** questions in Section A. Each question in Section A is worth 9 marks.
Answer **two** questions from Section B. Each question in Section B is worth 20 marks.

Write the number of the question at the beginning of each answer.

SECTION A

Answer **all** questions in section A.

(1) Could there be:

(a) a valid argument with a true conclusion but a false premise?
(b) a valid argument with only false premises and a false conclusion?
(c) a sound argument whose conclusion is a tautology?
(d) a sound argument with a contradiction as a premise?

If so, provide an example of such an argument. If not, explain why not.

(2) Use truth-tables (complete or partial) to assess the following:

(a) $A \vee B, B \vee C, \neg A \vDash B \wedge C$
(b) $(\neg A \leftrightarrow B) \vDash \neg(\neg A \leftrightarrow \neg B)$
(c) $\vDash (A \to B) \vee (B \to A)$

(3) Using the formal proof system from *forallx*, show that:
$\forall x(Fx \to Gx), \exists x(Fx \wedge Hx) \vdash \exists x(Gx \wedge Hx)$

(4) Provide examples of relations with the following properties:

(a) reflexive and symmetric but not transitive
(b) transitive and symmetric but not reflexive
(c) reflexive but neither symmetric nor transitive

(5) You roll two fair six-sided dice, once. Calculate the probability that:

(a) you roll 11.
(b) you roll 11, given that at least one of the dice showed a 6.
(c) you roll 11, given that both dice show the same number.

SECTION B

Answer any **two** questions from section B.

(6) Using the following symbolisation key:

Domain: people
D: $\_\_\_\__1$ is a drummer
B: $\_\_\_\__1$ is a bassist
L: $\_\_\_\__1$ likes $\_\_\_\__2$
a: Ali
b: Barker

symbolise all of the following English sentences as best you can in FOL. Comment on any difficulties you encounter:

(a) Ali likes Barker, and also other people.
(b) Every bassist likes a drummer.
(c) The drummer who likes Ali is not Barker.
(d) Provided Ali likes Barker, some bassist likes some drummer.
(e) Exactly two drummers other than Barker like Ali.

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(f) The drummer who likes Ali is not the bassist who likes Barker.
(g) Barker likes each of the three bassists.
(h) For every drummer who likes a bassist, some other drummer likes no one but Ali.
(i) Someone who is liked by nobody likes everyone.
(j) Someone likes all and only those who do not like themselves.

(7) Grange Knoll is a school with a population of 800 children and 200 adults. Sadly, 40 children and 40 adults in Grange Knoll have the flu. A member of the Grange Knoll is chosen at random; calculate the probability that:
(a) they have the flu.
(b) they have the flu, given that they are a child.
(c) they have the flu, given that they are an adult.
(d) they are a child, given that they have the flu.
(e) they are an adult, given that they do not have the flu.

A test has been developed, to determine whether or not someone has the flu. Among those who have the flu, the test delivers a positive verdict 95% of the time. Among those who do not have the flu, the test delivers a positive verdict 5% of the time. A member of Grange Knoll is chosen at random; calculate the probability that:
(f) they have the flu, given that the test delivered a positive verdict
(g) they have the flu, given both that they are a child and that the test delivered a positive verdict
(h) they are an adult, given that the test delivered a positive verdict

(8) Using the formal proof system from *forallx*, show each of the following:
(a) $\forall x \exists y(Rxy \lor Ryx), \forall x \neg Rmx \vdash \exists x Rxm$
(b) $\forall x(\exists y Lxy \to \forall z Lzx), Lab \vdash \forall x Lxx$
(c) $\forall x((Px \land \exists y Lyx) \to Dx), \forall x(Dx \to \neg \exists y Lyx) \vdash \forall x(Px \to \neg \exists y Lyx)$
(d) $\forall x(Lax \to \forall y(Lay \to x = y)), \neg Pc \vdash Lac \to \forall x(Lax \to \neg Px)$
(e) $\forall x(\neg Mx \lor Lax), \forall x(Cx \to Lax), \forall x(Mx \lor Cx) \vdash \forall x Lax$
(f) $\forall x(Lax \to x = a), \forall x(\exists y Lxy \to x = a) \vdash \forall x(\exists y Lyx \to x = a)$

(9) Attempt all parts of this question
(a) Let $A = \{\text{Algeria, Benin, Chad}\}$, $B = \{\text{Benin, Chad}\}$, $C = \{\text{Chad, Djibouti, Egypt}\}$, and $D = \{\text{Fiji}\}$. Calculate the members of each of the following sets:
(i) $(B - C) \cup D$
(ii) $(A \cap B) \cup (C \cap D)$
(iii) $A \times B$
(iv) $\{x : x \subseteq A \cap B\}$
(v) $\wp(B \cap C)$
(vi) $\wp(\wp(C \cap D))$
(vii) $(A - C) \times D$
(viii) $\wp(B - C) \times \wp(D)$
(b) Is there any set A such that $\wp(A) = \varnothing$? If so, give an example; if not, explain why not.
(c) Show that $A \setminus (C \setminus A) = A$, no matter what sets A and C are.

END OF PAPER