CONDITIONALS LECTURE 1

1. English appears to have two main types of conditional expression: **indicative** and **subjunctive** conditionals. 'If you leave at 9am you'll get there by 2pm' is an indicative conditional; 'If you *had* left yesterday afternoon you *would* have been there by now' is a subjunctive. The standard argument for distinguishing their semantics comes from the well-known pair due to E. Adams, which we are supposed to imagine as having been asserted by someone who is highly confident that Oswald killed Kenned and that he was acting alone.

(1.1) If Oswald didn't kill Kennedy then someone else did
(1.2) If Oswald hadn't killed Kennedy then someone else would have

Clearly in this situation (1.1) would be highly assertible but (1.2) not. There are some who have challenged this distinction (especially V. Dudman: see e.g. his paper in F. Jackson, ed., Conditionals) but most writers take it for granted; here it underlies the basic division in these lectures. Here I will write $p \rightarrow q$ for the indicative and $p > q$ for the subjunctive conditional.

2. When is an indicative conditional true? If the indicative conditional is a logical constant then it can only be the material conditional, but there are well-known arguments that its truth conditions cannot be those of the material conditional. For a simple one, consider the following, said of a six-sided die:

(2.1) If this die lands on an even number then it lands on 4

Instead of thinking directly about the *truth-conditions* of this $p \rightarrow q$ think about the *probability* that they obtain: whatever exactly probability is, it seems that there is a natural reading of 'Pr' on which $\mathrm{Pr}(p \rightarrow q) = 1/3$; but $\mathrm{Pr}(p \supset q) = 2/3$. So whatever proposition an indicative conditional is, it seems that it can't be the material conditional.

3. In fact what (2.1) suggests is that the probability of a conditional is given by its conditional probability: that is, the probability of its antecedent conditional on its consequent. More specifically: define: $\mathrm{Pr}(q|p) = \mathrm{Pr}(pq)\,/\,\mathrm{Pr}(p)$. Then the **Adams Thesis** says:

(3.1) $\mathrm{Pr}(p \rightarrow q) = \mathrm{Pr}(q|p)$

The probability of a conditional is just its conditional probability. Intuitively speaking this gets (2.1) right; note that (3.1) is inconsistent with the identification of an indicative conditional with the corresponding material conditional.

4. This is not all that can be said for the Adams thesis. We can think about probability as credence (degree of belief), so interpreted (3.1) says something intuitive about what it is to believe a conditional to a certain degree. More specifically, we can identify $\mathrm{Pr}(q|p)$ with the rate at which you would accept a bet on q that gets called off if p is false i.e. a conditional bet. What is the maximum x such that you would pay £x for a ticket that pays:

- £1 if the match is played and team Q wins
- Nothing if the match is played and team Q doesn't win
- £x if the match is not played

Letting p be the proposition that the match is played and q the proposition that team Q wins, it follows on the Ramsey-De-Finetti interpretation of probability as betting rates, that $x = \Pr(q|p)$. And it follows from the Adams thesis that this is also your confidence in the proposition $p \rightarrow q$: If the match is played then team Q will win. That is: $x = \Pr(p \rightarrow q)$.[1] Again, this is intuitively compelling.

5. A related but distinct advantage of the Adams thesis is that it explains the validity of the 'Ramsey test'. That idea, which Ramsey proposed in a separate paper, concerns how to tell whether you believe a conditional. He wrote: 'If two people are arguing ['If p then q'] and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q ... We can say that they are fixing their degrees of belief in q given p' (Ramsey 'Law and Causality' in his *Foundations of Mathematics* p. 143n). The connection is that the Adams thesis explains (because it entails the validity of) the Ramsey test, on the Bayesian assumption that when you learn a new proposition p your confidence in any proposition q should change from $\Pr(q)$ to $\Pr(q|p)$.

6. So there are plenty of reasons to accept the Adams thesis. Unfortunately there is also a really big reason not to accept it, which is **Lewis's Triviality Proof** (for the original version see his 'Probabilities of conditionals and conditional probabilities I', in his *Collected Philosophical Review* 85 (1976)). What Lewis shows is that there *is no* proposition such that its probability is always the probability of q given p. The proof has been refined since Lewis's initial publication of it.

7. We'll go over the details of Lewis's argument next time, but what is more important is being clear on the basic content of what he is saying. A proposition is just something that makes a condition on reality which reality either meets or fails to meet, so that at every possible world it is true or false. It is also, at least in Lewis's system, and object of belief and credence: you believe a proposition to a certain degree if you believe with that level of confidence that the world does meet or fail to meet that condition. What Lewis's argument shows is that if the Adams thesis were true then a conditional does not make a claim on reality, and it is not therefore something that you could believe with this or that degree of confidence.

8. For an analogy, imagine that each proposition is a subregion of the set of all possible worlds. And imagine that we can scatter dust across that region in such a way that the probability of a proposition p is just the proportion of all the dust that covers that region. Then what Lewis shows is that there is no region *r* – no set of worlds – such that however we scatter the dust, the proportion of dust in the region r is just the proportion of dust in the p region that is also in the q region.

[1] For the betting rate interpretation of probability see Ramsey, 'Truth and Probability' in his Philosophical Papers, ed. D. H. Mellor, CUP 1990; de Finetti B. (1992) Foresight: Its Logical Laws, Its Subjective Sources. In: Kotz S., Johnson N.L. (eds) Breakthroughs in Statistics. Springer Series in Statistics (Perspectives in Statistics). Springer, New York, NY. https://doi.org/10.1007/978-1-4612-0919-5_10