PHILOSOPHY TRIPOS Part IA

12:00 noon BST Friday 4 June 2021 -
12:00 noon BST Saturday 5 June 2021

Paper 5

FORMAL METHODS

*Answer **all** questions in Section A. Each question in Section A is worth 9 marks.*

*Answer **two** questions from Section B. Each question in Section B is worth 20 marks.*

Write the number of the question at the beginning of each answer

SECTION A

(1) Which of the following claims are true, and which are false? Briefly explain your answers.

(a) Every argument with a tautological premise is valid.

(b) If two sentences are provably equivalent, then they are tautologically equivalent.

(c) No sound argument has a false conclusion.

(2) Demonstrate the truth of the following claims, either by providing a truth table or counter-interpretation:

(a) $P \leftrightarrow Q, \neg(P \wedge Q) \vDash \neg Q$

(b) $P \rightarrow (Q \rightarrow R), \neg R, \neg Q \nvDash P$

(c) $\exists x \forall y(Fy \leftrightarrow x = y), \exists x\big(Gx \wedge \forall y(Gy \rightarrow x = y)\big) \nvDash \exists x \exists y(Fx \wedge Gy \wedge \neg x = y)$

(3) Prove the following using the proof system from *forallx: Cambridge*:

(a) $A \vee B, B \rightarrow C, A \rightarrow C \vdash C$

(b) $A \rightarrow B, \neg C \rightarrow \neg B \vdash A \rightarrow C$

(c) $\exists x Fx, \forall x(Fx \rightarrow Gx) \vdash \neg \forall x \neg Gx$

(4) Let *A* = {Boris, Jeremy} and *B* = {Jeremy, Jo, Nicola}. Give the members of the following:

(a) $\mathcal{P}(A) - A$

(b) $(A \cap B) \times A$

(c) $\mathcal{P}((A \cup B) \cap (A \cap B))$

(5) An urn contains 5 blue balls, 5 yellow balls, 4 red balls, and 6 white balls, and nothing else. Two draws are made from the urn at random without replacement. Calculate the probability of:

(a) drawing exactly one blue ball.

(b) drawing at least one yellow ball.

(c) drawing at least one yellow ball conditional on drawing exactly one blue ball.

SECTION B

(6) (a) Using the following symbolisation key:

Domain: people
a: Alfred
b: Bertrand
L: $\_\_\_\_\__1$ is a logician
P: $\_\_\_\_\__1$ is a philosopher
B: $\_\_\_\_\__1$ wrote a book with $\_\_\_\_\__2$

symbolise the following arguments in FOL, commenting on any difficulties or limitations.

(i) Alfred wrote a book with Bertrand only if he wrote a book with a logician, but all logicians are philosophers, so if Alfred didn't write a book with a philosopher, then he didn't write a book with Bertrand.

(ii) If Alfred wrote a book with Bertrand, then everyone did. And, certainly, someone wrote a book with Bertrand; so Bertrand must have written a book with himself!

(iii) Every philosopher has written a book with a logician, so there's a logician with whom every philosopher has collaborated.

(iv) It's not the case that: Alfred is a philosopher just in case he's a logician. So he's not a philosopher if, and only if, he's a logician.

(v) Bertrand must be a philosopher. After all, someone is a philosopher and Bertrand is the only philosopher.

(vi) Alfred wrote a book with Bertrand, and Alfred is both a philosopher and a logician. So there is at least one philosopher with whom Bertrand has written a book.

(b) For each of the arguments in part (a), decide whether they are valid or invalid.
For each valid argument, provide a proof using the system from *forallx: Cambridge.* For each invalid argument, provide a counter-interpretation.

(7) (a) Provide natural deduction proofs for the following:

(i) $\vdash \exists x\, x = x$

(ii) $\vdash \exists x(Fx \vee \neg Fx)$

(iii) $\forall x Fx \vdash \exists x Fx$

(b) Explain why the semantics of FOL justifies the claims made in part (a).

(c) Explain why we typically use natural deduction to prove *validity* and we typically use the semantics to prove *invalidity* in FOL.

(d) In TFL, if an argument is valid, then its conclusion can be proved from its premises. If we removed TND from the basic rules of inference, explain why this would no longer hold.

(e) In FOL, if a sentence is a theorem, then it is a logical truth. Give an example of a rule we could add to the deductive system of FOL so that this would no longer hold.

(f) What is wrong with the following semantic clause for the universal quantifier?

'$\forall \boldsymbol{x} \boldsymbol{A}(\ldots \boldsymbol{x} \ldots \boldsymbol{x} \ldots)$' is true in an interpretation iff '$\boldsymbol{A}(\ldots \boldsymbol{c} \ldots \boldsymbol{c} \ldots)$' is true for every name **c** in the language of FOL.

(8)

(a) A relation *R* is *serial on a set S* iff for every $x \in S$ there is some $y \in S$ such that $(x, y) \in R$.

(i) Is every relation which is reflexive on a set also serial on that set? Substantiate your answer.

(ii) Give an example of a relation which is serial but not reflexive on a set.

(iii) If a set contains exactly one member, must a serial relation on that set be reflexive on that set? Substantiate your answer.

(b) Can a relation be both reflexive and not symmetric on a single set? Substantiate your answer.

(c) Can a non-empty anti-symmetric relation on a set be anti-reflexive on that set? Substantiate your answer.

(d) A relation *R* is *sour on a set S* iff for every distinct $x, y \in S$, either $(x, y) \in R$ or $(y, x) \in R$. A relation *R* is *sweet on a set S* iff for every $x, y \in S$, there is some $z \in S$ such that $(x, z), (y, z) \in R$.

(i) Show that every relation which is reflexive and sour on a set is also sweet on that set.
(ii) Give an example of a relation which is reflexive and sweet on a set but not sour on that set.

(e) Answer each of the following and substantiate your answer in each case.

(i) If *R* and R' are both symmetric relations on a given set, is $R \cup R'$ also symmetric on that set?
(ii) If *R* and R' are both transitive relations on a given set, is $R \cup R'$ also transitive on that set?

(9) (a) Prove the following and justify each step of the proof:

$$\Pr(H|E) = \frac{\Pr(E|H)\Pr(H)}{\Pr(H)\Pr(E|H) + \Pr(\overline{H})\Pr(E|\overline{H})}$$

(b) 2 out of 9,000 Wonka Bars contain a golden ticket. There is a scanner for detecting whether a Wonka Bar contains a golden ticket. If the Wonka Bar contains a golden ticket, the scanner gives a positive verdict 90% of the time. If the Wonka Bar does not contain a golden ticket, the scanner gives a negative verdict 90% of the time. Calculate the probability of:

(i) the scanner giving a positive verdict when it scans a Wonka Bar.

(ii) a Wonka Bar containing a golden ticket, given the scanner gives a positive verdict when it scans the bar.

(c) In the game of extreme shuffle-cup there are 60 black cups, 20 white cups, 40 green cups, and 6 balls. Each of the 6 balls is placed randomly under a single cup, so that no cup covers more than one ball, as follows: 3 of the 6 balls are placed randomly under black cups, 1 of the 6 balls is placed randomly under a white cup, and 2 of the 6 balls are placed randomly under green cups. The cups are shuffled randomly. Calculate the probability of:

(i) a cup containing a ball, given that it is a black cup.

(ii) a cup being green, given that it contains a ball.

Steve is an experienced extreme shuffle-cup player who responds either ‘yes’ or ‘no’ to your questions. If a given cup contains a ball and you ask him whether it does he will answer ‘yes’ 80% of the time. If a given cup does not contain a ball and you ask him whether it does he will answer ‘no’ 90% of the time.

Calculate the probability of:

(i) a cup containing a ball given that you ask Steve whether it does and he answers ‘yes’.
(ii) the cup being white given that you ask Steve whether it contains a ball and he answers ‘no’.

END OF PAPER