PHT0/5

## UNIVERSITY OF CAMBRIDGE

## PHILOSOPHY TRIPOS, PART IA (PHT0/5)

Friday 03 June 2022 9.00am-11.00am

Paper 5

FORMAL METHODS

Answer **ALL** questions in Section A. Each question in Section A is worth 9 Marks.

Answer two questions from Section B. Each Question in Section B is worth 20 marks.

Write the number of the question at the beginning of each answer

# SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

# Calculator – students are permitted to bring an approved calculator

### **Stationary Requirements**

20 Page Answer Book x1 Rough Work pad

> You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

## Part IA Paper 5 (Formal Methods)

#### Section A (9 marks each)

- 1. Which of the following claims are true, and which are false? Briefly explain your answers.
  - (a) Any jointly tautologically consistent sentences admit of a valuation where all the sentences are false.
  - (b) Any jointly tautologically inconsistent sentences will remain tautologically inconsistent if any other sentence is added.
  - (c) If an argument's premises are jointly inconsistent with its conclusion, the argument must be invalid.
- 2. Determine whether the following claims are true or false. Where true, prove this by using the natural deduction system in *forallx:Cam*; where false, provide a truth-table.
  - (a)  $A \wedge \neg B$  is tautologically equivalent to  $\neg A \vee B$
  - (b)  $A \to (\neg B \to \neg A)$  is tautologically equivalent to  $A \to B$
  - (c)  $A \to (B \leftrightarrow C)$  is tautologically equivalent to  $(B \to C) \land (\neg A \lor B \lor \neg C)$
- 3. Use the following symbolisation key for this question:
  - Domain: all species of animals
  - $S: -_1$  is covered in spots
  - $M: -_1$  is a species of monkey
  - $H: -_1$  hunts  $-_2$

Give symbolisations of the following sentences:

- (a) Everything is hunted by something.
- (b) There are exactly two species which are not covered in spots.
- (c) All monkey species are hunters.
- Offer natural-language versions of the following claims:
- (d)  $\exists x \forall y (Hxy \rightarrow Sy)$
- (e)  $\exists x(Mx \land Sx \land \forall y((My \land Sy) \to y = x))$
- (f)  $\forall x((Sx \land \exists y(My \land Hxy)) \rightarrow \exists y(\neg My \land Hxy))$
- 4. If  $A = \{1, 2, 3\}$  and  $B = \{3, 4, 5\}$  then write down the members of the following sets:
  - (a)  $A \cap B$
  - (b)  $A \cup B$
  - (c) A (A B)
  - (d)  $\varnothing B$
  - (e)  $A \cap \wp(A)$
  - (f)  $\wp(\wp(A \cap B))$
- 5. Give examples of relations with the following properties:
  - (a) Reflexive and symmetric but not transitive
  - (b) Symmetric and transitive but not reflexive
  - (c) Transitive and reflexive but not symmetric

#### Section B (20 marks each)

- 1. (a) Consider the interpretation whose domain consists of Vincent, Mary, Paul, and Fanette, all of whom are distinct from one another; and where the predicate 'R' is to be true of (and only of)
  - (Vincent, Fanette), (Paul, Fanette), (Mary, Vincent), (Fanette, Paul)

Determine the truth values of each of the following sentences in this interpretation. You do not need to explain your answers.

$$\forall x \forall y \forall z \forall w (x = y \lor x = z \lor x = w \lor y = z \lor y = w \lor z = w)$$
(A)

$$\forall x \forall y \forall z (Rxy \to (Ryz \to Rxz)) \tag{B}$$

$$\forall x \exists y R x y \tag{C}$$

$$\forall x \neg Rxx$$
 (D)

- (b) Demonstrate that the above sentences (A)–(D) are not jointly consistent. You may do so by reasoning about interpretations, or (if you prefer) by the use of the natural deduction system in *forallx:Cam*.
- (c) Show that any three out of the four sentences are jointly consistent: that is, show that
  - i. sentences (A), (B), and (C) are consistent;
  - ii. sentences (B), (C), and (D) are consistent;

iii. sentences (A), (B), and (D) are consistent; and

- iv. sentences (A), (C), and (D) are consistent.
- (d) In TFL, one can verify that some sentences are not jointly consistent by considering finitely many valuations (e.g. in the form of a truth-table). Briefly explain why considering finitely many interpretations is not, in general, sufficient to verify that some FOL-sentences are not jointly consistent.
- 2. (a) Prove the following using the natural deduction system in *forallx:Cam*:

i. 
$$\forall x \forall y (Px \to (Qy \to \neg Rxyx)) \vdash \forall x \forall y (Rxyx \to (Qy \to \neg Px))$$
  
ii.  $\forall x (Px \to \exists y (Ryx \land Py)), \forall x \forall y \forall z (Ryx \to (Rzy \to \neg Pz)) \vdash \neg \forall x Px$ 

(b) In the rule  $\forall I$ ,

$$\begin{array}{c|c} m & \mathbf{A}(\dots\mathbf{c}\dots\mathbf{c}\dots) \\ & \forall \mathbf{x} \mathbf{A}(\dots\mathbf{x}\dots\mathbf{x}\dots) & \forall \mathbf{I}, m \end{array}$$

why do we insist that the name  $\mathbf{c}$  must not occur in any undischarged assumption? Provide an example of an invalid argument that would be provable without this constraint.

(c) In the rule  $\exists E$ ,

$$\begin{array}{c|c} m \\ i \\ j \\ B \\ \end{array} \begin{array}{c} \exists \mathbf{x} \mathbf{A}(\dots \mathbf{x} \dots \mathbf{x} \dots) \\ \hline \mathbf{A}(\dots \mathbf{c} \dots \mathbf{c} \dots) \\ \hline \mathbf{B} \\ \end{array} \\ \exists \mathbf{E}, \ m, \ i-j \end{array}$$

why do we insist that  $\mathbf{c}$  must neither occur in  $\exists \mathbf{x} \mathbf{A}(\dots \mathbf{x} \dots \mathbf{x} \dots)$ , nor in  $\mathbf{B}$ ? For each constraint, provide an example of an invalid argument that would be provable without that constraint.

- 3. For each of the following relations, say
  - i. whether it is reflexive,
  - ii. whether it is symmetric, and
  - iii. whether it is transitive

on the domain of all people alive today. (Assume that some people are wise and some are foolish, that some people are neither wise nor foolish, and that no-one is both wise and foolish; that some people live in Saffron Walden and others do not; and that no one is his own brother.)

- (a) x is wise unless y is foolish
- (b) x and y are married
- (c) x and y are brothers
- (d) x is at least 2 years older than y
- (e) x is at least 200 years older than y
- (f) x is Boris Johnson iff y is Keir Starmer
- (g) Either x has the same first name as y or x has the same surname as y
- (h) Exactly one individual in the set  $\{x, y\}$  lives in Saffron Walden
- 4. You draw three cards without replacement from a pack of ten cards numbered 1–10. What is the probability that:
  - (a) The first is higher than the second?
  - (b) All three cards add up to 8?
  - (c) The second is even, given that the first and the third are even?
  - (d) The third is higher than the second, given that the second is higher than the first?

#### END OF PAPER