

PHILOSOPHY TRIPOS, PART II

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Wednesday 08 June 2022

1.30pm-4.30pm

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Paper 7

MATHEMATICAL LOGIC

Answer **three** questions only.

*Write the number of the question at the beginning of each answer.  
If you are answering an either/or question, indicate the letter as well.*

**STATIONERY REQUIREMENTS**

*20 Page Answer Book x 1  
Rough Work Pad*

You may not start to read the questions  
printed on the subsequent pages of this  
question paper until instructed that you  
may do so by the Invigilator

1. EITHER: (a) 'The iterative conception of set states that sets are formed after their members.' How are we to understand this temporal language? What is its significance for the status of the iterative conception?

OR: (b) 'There is no good reason to accept the axioms of replacement. They are not motivated by the iterative conception and their benefits can be had without it.' Discuss.

2. Outline a theory of transfinite ordinals and illustrate ways in which it differs from the finite case.

3. Prove Cantor's theorem. Explain the sense in which your proof is impredicative.

4. EITHER: (a) What is meant by 'aleph-null categoricity'? Prove that first-order complete arithmetic is not aleph-null categorical. How would your proof break down when applied to second-order Peano arithmetic?

OR: (b) 'Second-order theories are categorical, and so capture mathematical practice. Hence second-order logic has a better claim to be logic than first-order logic.' Discuss.

5. What is an effectively enumerable set? How can an effectively enumerable set fail to be decidable? Explain which of the following comprise effectively enumerable sets:

- (i) the tautologies of the propositional calculus
- (ii) the sentences of first-order logic with identity that are not logical truths
- (iii) the logical truths of second-order logic
- (iv) the theorems of first-order complete arithmetic.

6. Outline a proof of the completeness of first-order predicate calculus without identity. Explain how to extend your proof to the case with identity.

7. Show, for a suitable notion of 'expressive' (which you should define), that any axiomatic theory in a sufficiently expressive language is either unsound or incomplete.

8. Can the notion of computability be given a precise mathematical definition? If so, how? If not, why not?

9. Was Hilbert's programme a *total* failure?

10. In what sense, if any, is 'countability' a relative notion?

**END OF PAPER**