

PHILOSOPHY TRIPOS Part II

12:00 noon BST Friday 11 June 2021 –
12:00 noon BST Saturday 12 June 2021

Paper 7

MATHEMATICAL LOGIC

Answer **three** questions only.

Write the number of the question at the beginning of each answer.

For each question do not write more than **2000** words.

1. 'It is impermissible to use the language of second-order logic to formalise discourse about certain sorts of objects, such as sets or ordinals, in case there is no set to which all objects of that sort belong.' Discuss.
2. Explain the role of Henkin constants (also known as witnesses) in the proof of the completeness of first-order predicate logic without identity. What further complexities are introduced in proving completeness for the system with identity? What role does mathematical induction play in these proofs?
3. What is meant by a 'non-standard model'? Do such models have any philosophical significance?
4. How do cardinals and ordinals differ?
5. What is the iterative conception of set? Which axioms of ZFC does it justify?
6. In what sense, if any, does set theory provide a foundation for ordinary mathematics?
7. 'An algorithm is a procedure for which we can give exact instructions for carrying it out.' Discuss.
8. "No formal system for arithmetic can prove its own consistency." Is this correct? If not, why not?
9. 'PA can prove its own consistency. This is because PA can capture ZFC as a deductive system and ZFC can prove the consistency of PA.' What is wrong with this reasoning? How does your answer bear on the question of whether minds are machines?
10. How far can Hilbert's programme be taken?

END OF PAPER