The birth of analytic philosophy*

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Analytic philosophy was, at its birth, an attempt to escape from an earlier tradition, that of Kant, and the first battleground was mathematics. Kant had claimed that mathematics is grounded neither in experience nor in logic but in the spatio-temporal structure which we ourselves impose on experience. First Frege tried to refute Kant's account in the case of arithmetic by showing that it could be derived from logic; then Russell extended the project to the whole of mathematics. Both failed, but in addressing the problems which the project generated they founded what is nowadays known as analytic philosophy or, perhaps more appropriately, as the analytic method in philosophy. What this brief summary masks, however, is that it is far from easy to say what the analytic method in philosophy amounts to. By tracing the outlines of the moment when it was born we shall here try to identify some of its distinctive features.

1 Frege

1.1 Begriffsschrift

In 1879 Frege published a short book called *Begriffsschrift* (Conceptual Notation). What this book contains might nowadays be described as a formalization of the predicate calculus, the part of logic dealing with quantification. Frege's aim in trying to formalize logic was to codify the laws not of thought but of truth. He was commendably clear from the start, that is to say, that logic is not a branch of psychology. Logic consists of the laws to which our reasoning ought to adhere if it is to aim at the truth, not of those to which our reasoning does in fact adhere. There are certainly errors in reasoning which most people are inclined to make, but Frege's point is that it is indeed appropriate to describe these as *errors*. He regarded it as possible for there to be a form of reasoning which all of us have always been inclined to accept but which is, in some way not yet detected by any of us, a mistake.

Frege was certainly not the first to formalize part of logic: that was Aristotle. And 200 years before Frege Leibniz had even had the ambition of developing a formal system that would reduce reasoning to a mechanical process like arithmetic. But there is nothing in Leibniz's surviving writings to show that he carried forward this project very far. More recently, however, British logicians such as Boole, Jevons and Venn had made significant progress: Boole had invented a notation for expressing the logical operations of negation, conjunction and (exclusive) disjunction, and he had discovered that the logical rules which propositions involving these operations obey are strikingly similar to those of elementary arithmetic; Jevons had designed a 'logical piano', a machine which could solve problems in Boolean logic with impressive speed and accuracy.

This work can thus be seen as a working out, for one part of logic, of Leibniz's ambition. Once a proposition has been expressed in Boolean notation, it can be transformed by means of quasi-arithmetical rules into a simpler but logically equivalent form, in a manner that is quite analogous to the algebraic manipulations of elementary arithmetic. Boole's method has turned out to have widespread practical applications: it can be used, for instance, to simplify electrical circuits and computer programmes.

Nonetheless, what Boole was doing was to develop a technique within the scope of logic in the sense in which it had been understood since the time of Aristotle. What distinguishes Frege's work from Boole's is that he advanced into quite new territory by inventing a notation for quantifiers and variables. There is no doubt a sense in which the idea of quantifiers and variables was already 'in the air' in 1879. It is at any rate striking that Peirce (1885) invented his own notation for quantifiers and variables independently at almost exactly the same time as Frege. But it was for Peirce only a notational device, not in itself a tool for reasoning, and he did not develop the idea with anything like Frege's philosophical subtlety. One reason for this, no doubt, is that Peirce was working much more in the algebraic tradition of Boole and Jevons. It did eventually turn out that Boole's idea of treating reasoning as a form of algebraic manipulation can be generalized to encompass reasoning that involves quantifiers: the notion that plays the analogous role to that of a Boolean algebra is called a 'cylindric algebra'. But when the theory of cylindric algebras was worked out in the 1930s by Tarski, it quickly became apparent why no-one had thought of it before: the theory is, at least by comparison to the method involving rules of inference, inelegant and unintuitive.

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What is important about Frege's work, in comparison to Boole's, is thus that it enlarged the scope of formal logic decisively. It would be an exaggeration to say that Frege's was the first major advance in logic since Aristotle, but it would not be wholly false either. The mediaevals had been aware that what can be shoe-horned into the form of the Aristotelian syllogism by no means exhausts the forms of reasoning that are to be counted as valid, and they had therefore striven to extend the scope of formal logic accordingly. But they had done so piecemeal: the decisive advance had always eluded them.

The reason Frege's invention of polyadic predicate calculus counts as decisive is one that received precise expression only half a century later when Church and Turing showed in 1936 (independently of one another) that it is not mechanically decidable which arguments involving polyadic quantification are logically valid. By contrast the corresponding problem for arguments involving only monadic quantification (or, indeed, for the Aristotelian syllogistic) *is* mechanically decidable. Church and Turing's discovery marks a major step in logic, since by showing for the first time that there are problems in logic which cannot be solved mechanically it demonstrated a disanalogy between logic and elementary arithmetic and hence showed that there must be some limits to Leibniz's dream of a mechanical calculus to take over the task of reasoning.

Although Frege never knew of this limitative result, he seems to have had a sense from the outset of the remarkable power of the method he had invented:

> Pure thought, irrespective of any content given by the senses or even by an intuition a priori, can, solely from the content that results from its own constitution, bring forth judgments that at first sight appear to be possible only on the basis of some intuition. (Frege 1879, §23)

This remark of Frege's about the power of reasoning that involves polyadic quantification is in marked contrast to what earlier philosophers had said about Aristotelian logic and its mediaeval accretions. When Descartes, for example, said of logic that 'its syllogisms and most of its other instructions serve to explain to others what one already knows' (*Discourse on Method*, part 2), it was syllogistic – therefore decidable – logic he had in mind. And Kant, in presenting the central task of the first Critique as that of explaining how synthetic a priori knowledge is possible, had taken arithmetic as his first and best example of a domain of synthetic truths. If what he meant by an analytic truth was anything that can be deduced from explicit definitions by syllogistic logic, then what is analytic is in an important sense trivial. If, on the other hand, we enlarge the scope of the analytic to include what can be deduced by means of polyadic logic, what then remains of Kant's claim that arithmetic is synthetic (and hence, according to Kant, dependent in some way on the spatio-temporal structure of the world as we experience it)?

1.2 Grundlagen

Frege was not the only person interested in this question. Dedekind's beautiful treatise, *Was sind und was sollen die Zahlen?* (1888), also attempts to show, contrary to Kant, that arithmetic is independent of space and time. There are three stages involved in establishing this claim. The first is to characterize the natural numbers in axiomatic terms and show that the familiar arithmetical properties follow logically form these axioms. The second is to show that there exists a structure satisfying these axioms. The third is to abstract from the particular properties of the structure used in the second stage, so as to identify the natural numbers as a new structure satisfying the axioms. Dedekind's execution of the first of these three stages may be counted a complete success: the axioms he identified (which are nowadays called Peano's axioms) do have as logical consequences all of the truths of arithmetic. But the second and third stages are more problematic. In order to achieve the second part of the programme Dedekind found that he needed to assume the so-called axiom of infinity, which asserts that there exist infinitely many objects. Dedekind thought he could prove this axiom, but for his proof to be regarded as correct we would at the very least have to widen the scope of logic even further than Frege had done, since what he proves is at best that the realm of thoughts that are available to us as reasoning beings is infinite. And for the third stage of the programme Dedekind appealed to a sort of creative abstraction that has seemed obscure to many later writers.

Frege's aim in *Die Grundlagen der Arithmetik* (1884) was the same as Dedekind's – to show that arithmetic is independent of space and time – and the shape of his approach was also the same: first he identified an axiomatic base from which the properties of the natural numbers could be deduced, then he tried to show logically that there exist objects satisfying these axioms, and finally he needed a principled reason to ignore whatever properties the objects chosen in the second stage may have that do not follow from the axiomatic characterization of them identified in the first stage.

But although the three stages of the programme were the same for Frege as for Dedekind, how he executed them differed significantly. In the first place Frege's axiomatic characterization of the natural numbers treated them as finite cardinal numbers and characterized cardinal numbers by means of the principle that the cardinals of two concepts *F* and *G* are equal if and only if there exists a one-to-one correlation of the *F*s with the *G*s (or, as is sometimes said, if *F* and *G* are 'equinumerous'). This equivalence, known in the recent literature on the topic (with tenuous historical licence) as 'Hume's Principle', can be used to derive the properties of the natural numbers in much the same way as Peano's axioms can. For the second stage of the programme, showing that there are objects satisfying Hume's Principle, Frege made use of the notion of the extension of a concept, i.e. a sort of logical object associated with a concept in such a way that two concepts have the same extension just in case they have the same objects falling under them. Frege defined the number of *F*s to be the extension of the (second-order) concept under which fall all those concepts equinumerous with the given concept *F*.

Having defined numbers in terms of extensions in this way, Frege needed some account of why the extra properties numbers acquire accidentally as a consequence of the definition can be ignored. But what Frege's account was is somewhat hazy. It is plain that he thought some role was played by the 'context principle', the methodological principle that it is only in the context of a sentence that words mean anything. The importance he placed on this principle is shown by the fact that he mentioned it in both the introduction and the conclusion to the *Grundlagen* as well as in the text, but it is less easy to see what this importance amounts to.

It sometimes seems, indeed, as if the importance of the context principle may lie not so much in Frege's use of it but in the significance it has been given subsequently by Frege's most noted commentator, Michael Dummett. According to Dummett, Frege's enunciation of the context principle marks a fundamental shift in philosophy, the so-called 'linguistic turn', of comparable significance to Kant's Copernican turn a century earlier.

The puzzle, though, is to see what role the context principle is supposed to play in Frege's account of numbers. If he had sought to treat Hume's Principle as a contextual definition of numbers, that role would be clear enough: the context principle seems designed precisely to allay any concern one might have that a contextual definition does not say what the term it introduces refers to but only gives us the meaning of whole sentences in which the term occurs. But Frege rejects the idea of treating Hume's Principle as a contextual definition of numbers because, while it settles the truth conditions of some of the identity statements in which number terms can occur, it does not settle them all. (Most famously, to use Frege's 'crude example', it does not settle whether Julius Caesar is a natural number.) Instead, as we have seen, Frege treats Hume's Principle only as a contextual *constraint* – a condition, that is to say, that any definition of natural numbers must satisfy if it is to be regarded as correct. But if we end up giving an explicit definition of numbers and then showing that numbers so defined do indeed satisfy the constraint, it is not at all clear what role is left for the context principle to play.

A further (and, as it was to turn out, much worse) problem for Frege was that the explicit definition of numbers that he settled on defined them in terms of the notion of the extension of a concept. But what is that? The best that he could be said to have achieved by the end of the *Grundlagen* was to reduce the problem he started with, of explaining how numbers are given to us, to the rather similar question of how extensions of concepts are given to us.

The similarity between the problems, as Frege thought of them, is indeed rather more than superficial. For Hume's Principle, the contextual specification of the identity conditions for numbers, has the form of an abstraction principle, which is to say that it asserts the logical equivalence of, on the one hand, an identity between two terms (in this case number terms) and, on the other, the holding of an equivalence relation (in this case equinumerosity) between the relevant concepts. But note now that the explanation we gave of the notion of the extension of a concept – that concepts have the same extension just in case the same objects fall under them – is an abstraction principle too. If the Julius Caesar problem puts paid to the idea of introducing numbers by means of the first abstraction principle, does it not also put paid to the idea of introducing extensions by means of the second?

This is a question Frege never satisfactorily answered. In the *Grundlagen* he did not even address it, mentioning only (in a footnote) that he would 'assume it is known what the extension of a concept is' (§68). Plainly a little more needs to be said, but when he came to say it, in *Grundgesetze der Arithmetik* (1893-1903), he confined himself to treating the notion of an extension within the formal language of the *Begriffsschrift*. In that language he does indeed introduce extensions by means of the abstraction principle just mentioned (which he calls 'Basic Law V'),[1] but he does not have to address the Julius Caesar problem because the formal language he is dealing with does not have terms for referring to Roman emperors.

It is plain that this is only a deferral of the problem, not a solution. Frege was clear, after all, that any satisfactory account of arithmetic would have to explain its applicability to the world, and he was

[1] Basic Law V is actually somewhat more general, but the extra generality is irrelevant to the point under discussion here.

scathing about the failure of formalism to deal with just this point. So at some point he would have to expand the formal language to encompass terms for Roman emperors, so that they could be counted, and he would have to do so in such a way as to settle the question whether Julius Caesar is a natural number (or, indeed, the extension of a concept).

1.3 Sense and reference

What was appealing to Frege about abstraction principles such as Hume's Principle or Basic Law V was, as we have seen, partly the validation which he somehow thought they receive from the context principle. But it also lay in his belief that they are in some weak sense logical. Just what that weak sense is, however, Frege was never able to say precisely. Indeed he granted that Basic Law V was more open to doubt than the other axioms of his theory. Nonetheless, he remained attracted to the thought, first enunciated in the *Grundlagen*, that the left hand side of an abstraction principle, which expresses an identity between the objects the principle seeks to introduce, is somehow a recarving of the content of the relation of equivalence between concepts which occurs on the right hand side.

The difficulty, then, is to say what the notion of content is which can give substance to this metaphor of recarving. When he wrote the *Grundlagen*, Frege had only a very coarse-grained theory of content to offer, according to which any two logically equivalent propositions have the same content. By the time of *Grundgesetze*, however, Frege had elaborated the theory of sense and reference for which he is now famous.

There is nothing deep, of course, in the distinction between a sign and the thing it signifies, nor in the distinction between both of these and the ideas I attach to a sign when I use it. What goes deeper is the claim that if we are to have a satisfying account of language's ability to communicate thoughts from speaker to listener we must appeal to yet a fourth element – what Frege calls sense.

The interest of Frege's notion of sense lies in two features of it. First, senses are abstract. Since the sense of an expression is what it is that is communicated from speaker to hearer, it must be possible for each of us to grasp it and it cannot, therefore, be something private to either of us, as an idea is. So a sense is not a mental entity. But neither, plainly, is it physical. It therefore inhabits what Frege calls a third realm,[2] defined negatively, of elements that are neither physical nor mental. (This alone, of course, has been enough to make many 20th century philosophers treat the notion with deep suspicion.)

Second, it is not just names like 'Hesperus' and 'Phosphorus' that have sense. The thought expressed by a whole sentence is a sense for Frege, and it is somehow composed out of the senses of the subsentential expressions that make up the sentence.[3] The theory is, that is to say, *uniform* in attributing sense to the meaningful elements of language: no linguistic item, for Frege, latches onto the world directly, but the reference of each is mediated by its sense, which is the mode by which the linguistic item presents the object it is supposed to refer to.

Both these aspects of Frege's theory are problematic. Quite apart from any suspicion some might have of abstract entities, it is hard to get a stable grasp of the notion of sense Frege required: a notion, namely, that is finely grained enough to distinguish the sense of 'Hesperus' from that of 'Phosphorus' (which it must if it is to explain why I can learn something about astronomy when you tell me that Hesperus = Phosphorus); and yet not so fine that it distinguishes the sense I, ignorant of astronomy, attach to the word 'Hesperus' from the sense you, who know much more about the planets, do (since if it does, the sense cannot be what is communicated when you tell me). And the compositionality of sense is puzzling too. It is certainly puzzling what sort of compositionality could make it the case that the two sides of an abstraction principle have the same sense. But even if we prescind from that and agree not to regard Frege's notion of sense as an attempt to legitimate this aspect of his project of using an abstraction principle to ground arithmetic, it remains puzzling what sort of composition is supposed to be at work.

[2] The expression 'ein drittes Reich' did not when Frege used it in (1918) have all the connotations which it later acquired.

[3] Frege also thought that the notion of reference could, parallel with sense, be given a treatment that is uniform for sentences and the expressions that make them up, so that a sentence has a reference, namely its truth value, in just the way that a name has reference, namely the object it names. This element of Frege's theory is clearly wrong, as Wittgenstein (1922, 4.063) showed.

2 Moore and Russell

2.1 Objective propositions

The second strand in the birth of analytic philosophy began in 1898. Russell later described it as having been born in conversations between him and Moore. What is clear at any rate is that the first publications that bear witness to it are Moore's articles on 'The nature of judgment' and 'The refutation of idealism'.

The overall shape of the revolution is clear: Moore thought that by conceiving of propositions as objective complex entities he could resist the temptations of idealism.

Once it is definitely recognized that the proposition is to denote not a belief (in the psychological sense), it seems plain that it differs in no respect from the reality to which it is supposed merely to correspond, i.e. the truth that *I exist* differs in no respect from the corresponding reality *my existence*. ('Truth' in Baldwin 1901)

At the centre of the project, in other words, was what would now be called an identity theory of truth. But if the overall shape of the project is clear, the details are not. Although 'The nature of judgment' is written in a crisp style that is in marked contrast to the narcoleptic pedantry of Moore's later work, it is nonetheless difficult to determine exactly what its arguments are. The targets of Moore's criticism are broadly spread: although it is Bradley's post-Hegelian denial that absolute truth is ever attainable which is the principal target, at times Berkeley's view that *esse est percipi* or Kant's view that the relations the objects of experience bear to one another are supplied by the mind are also attacked.

Moore's conception of a proposition is embodied in two central doctrines. The first is that the entities of which a proposition is composed (which he calls 'concepts') are themselves the items the proposition is about. He opposes this to Bradley's view that when I have an idea of something, that thing is itself part of the idea. This opposition is plainly not exhaustive of the possibilities, but once he had disposed (no doubt rightly) of Bradley's view, Moore seems to have seen no need of an argument for his own. Nonetheless, the doctrine is central to the refutation of idealism as Moore conceives of it. Propositions are the objects of judgment, and the concepts that make up the proposition are therefore part of what we judge, but the view is nonetheless realist because this is 'no definition of them'; 'it is indifferent to their nature,' he says, 'whether anyone thinks them or not.' (Moore 1899, p. 4) Concepts are, that is to say, objective entities.

The second central doctrine is that there are no internal relations between concepts – no relations between concepts that are part of the nature of the concepts related. What it is for a proposition to be true is just for the concepts it is composed of to be externally related to each other in a certain way. Once again, the main target is Bradley, who had denied that external relations are ever real. If knowledge is conceived of as an internal relation between the knower and the proposition known, the mere act of coming to know a proposition will alter it, since the property it now has of being known is internal to it and therefore makes it different from what it was before I knew it. For Bradley, therefore, no judgment is ever wholly true: judgment is inherently distorting. For Moore, on the other hand, the act of judgment relates a proposition to the judging subject only externally and does not thereby alter what is judged. But it is much less clear why in opposing Bradley's view Moore should have gone to the opposite extreme and said that there are *no* internal relations between concepts at all. And, as in the case of the first doctrine, Moore seems (at this stage at least) to have been oblivious to the need for an argument.

2.2 The Principles of Mathematics

The doctrine that there are no internal relations between concepts runs into an obvious difficulty in the case of identity statements. If the identity 'a=a' expresses anything about a, a relation between a and itself, it seems clear that this must be internal. So if there are no internal relations, we are forced to conclude that it does not express anything at all. This is perhaps not so bad in itself, but we shall need to say something about the identity 'Hesperus=Phosphorus', which, apparently at least, expresses genuine astronomical information. And a lot more will have to be said about arithmetic, in which apparently informative identity statements (such as '7+5=12') play such a central role.

The work in which this was attempted was Russell's *Principles of Mathematics* (1903). To modern readers (of whom there are not as many as one might expect, given its place in the history of the subject) this comes across as a transitional work: it contains extended passages which we can recognize as analytical philosophy in quite the modern sense, but these are juxtaposed to passages written in a style that strikes us as wholly antiquated introducing bizarrely elaborate classifications for no apparent reason that develop into an architectonic of almost Kantian complexity. In this regard Russell's book stands in interesting contrast to Frege's *Grundlagen*: there are indeed occasional longueurs in this book, arising in the main when Frege targets errors that we are no longer tempted to make, but the arguments Frege uses to dispose of them do not strike us as obsolete.

Russell's main purpose in writing the *Principles* was to make plausible a version of what is now called *logicism*: he wished to generalize to the whole of mathematics Frege's more limited claim that arithmetic is part of logic. Central to this project, as Russell now conceived it, was his adoption of Moore's conception of a proposition as containing the parts of the world it is about. But Russell now amended this conception by adding to it the notion of a denoting concept. A denoting concept is what one might call an 'aboutness shifter' (Makin 1995): its task is to enable a proposition to be about something else that is not itself part of the proposition. On Moore's view the proposition expressed by the sentence 'I met John' contains me, John and the universal *meeting*. What is expressed by 'I met a man' similarly contains me, meeting, and a third element expressed by the phrase 'a man'. But what is this third element? It cannot be any particular man, since it is just the same proposition whichever man it was that I actually met.

> The proposition is not about *a man*: this is a concept which does not walk the streets, but lives in the shadowy limbo of the logic-books. What I met was a thing, not a concept, an actual man with a tailor and a bank-account or a public-house and a drunken wife. (Russell 1903, §56)

Yet there must be *some* connection between the man with the bank-account and the propositional component in question. In the *Principles* Russell calls the propositional component a *denoting concept* and the relation it has to the man that of *denoting*. 'A concept *denotes* when, if it occurs in a proposition, the proposition is not *about* the concept but about a term connected in a certain peculiar way with the concept.' (Russell 1903, §56)

Russell seizes on denoting as the central element in his account of mathematics.

> The concept *all numbers*, though not itself infinitely complex, yet denotes an infinitely complex object. This is the inmost secret of our power to deal with infinity. An infinitely complex concept, though there may be such, can certainly not be manipulated by the human intelligence; but infinite collections, owing to the notion of denoting, can be manipulated without introducing any concepts of infinite complexity. (Russell 1903, §72)

A proposition about all numbers therefore does not itself contain all numbers but only a concept which denotes all numbers.

2.3 On denoting

In 1901, when Russell already had a complete draft of the *Principles of Mathematics*, he discovered the famous paradox which bears his name. He showed, that is to say, that the denoting concept, the class of all classes which do not belong to themselves, does not denote anything (since if it did, the class it denoted would belong to itself if and only if it did not belong to itself, which is absurd).

The paradox had already been discovered by the mathematician Zermelo at Göttingen a couple of years earlier (and other somewhat similar paradoxes were known to Cantor). What is significant about its rediscovery by Russell is the manner in which the problem it raises now affected philosophy. The most immediate effect of the paradox on Russell was that it made him focus his attention on those denoting concepts (such as, most famously, that of the present king of France) which do not denote anything. The point, of course, is not that he had until then been unaware that according to his theory there would have to be such concepts, but only that the paradox showed him the need to gain a better understanding of how they function. Russell had said that a proposition in which a denoting concept occurs 'is not about the concept but about a term connected in a certain peculiar way with the concept'. If the term in question does not exist, the way in which it is connected with the concept will indeed be peculiar.

But the moment of revelation for Russell came when he saw that the relationship is peculiar even when the term *does* exist. For if there is a relationship between the concept and the thing it denotes, there will be a true proposition expressing that relationship, and this true proposition will be about the concept. But a denoting concept, let us recall, is defined as one whose job is to occur in a proposition but to point at something else which the proposition is about. So how can *any* proposition be about the denoting concept itself? What sort of entity should occur in a proposition in order for the proposition to be about, say, the denoting concept expressed by the phrase 'the first line of Gray's Elegy'? Not, certainly, the denoting concept itself, since if it is doing its aboutness-shifting job properly, it will ensure that the proposition ends up being not about the concept but about what it denotes, i.e. about the sentence 'The curfew tolls the knell of parting day'. Nor, clearly, is it any use to put in the proposition the denoting concept 'the meaning of the first line of Gray's Elegy', since that would make the proposition be about the meaning of the sentence 'The curfew tolls the knell of parting day', which again is not what we want.

Up to this point there is something that is apt to strike the reader as puzzling. The argument is supposed to show that there can be no informative proposition about the concept expressed by the phrase 'the first line of Gray's Elegy'. Yet this last sentence seems to express a proposition that is about just this concept. Russell has to say that it is not what he wants. Why? At this point he introduces a further constraint. The relationship between a concept and its denotation (if any) is not, he says, 'linguistic through the phrase'. Concepts exist, he evidently thinks, whether or not we choose to devise means to express them in language. So the relationship between the concept and its denotation exists independent of language and hence so does the proposition expressing it. So any sentence in which a linguistic item, such as the phrase 'the first line of Gray's Elegy', is mentioned (rather than used) cannot be what we are after since the proposition it expresses will be about language whereas the proposition we are trying to express would, if it existed, be independent of language.

It is a staple of undergraduate essays on Russell's theory of descriptions to point out that it deals with the case of definite descriptions which do not refer to anything, but this, while true, was only ever part of the point. Russell's earlier theory of denoting had of course recognised that there are denoting phrases which do not denote anything. There is certainly in such cases a puzzle about the role of the corresponding denoting concept: if a denoting concept is thought of as a sort of pointer, a denoting concept that does not denote anything is a pointer pointing at nothing. But Russell's objection to the theory applies just as much in the case of denoting concepts that do denote something.

The argument we have just described (which is always known as the Gray's Elegy argument because of the example he uses to make the point) led Russell to reject the theory of denoting he had put forward in the *Principles*. What he replaced it with was an account according to which the true structure of the proposition a sentence expresses is to be revealed by translating it into the predicate calculus with identity. The sentence 'I met a man', for instance, might be translated as $(\exists x)(Mx \; \& \; Rax)$, where '$Mx$' means that x has the property of manhood, 'Rxy' means that x met y and 'a' denotes me. (In words: there is someone I met who has the property of manhood.) The denoting phrase 'a man' has disappeared, to be replaced by the notation of quantifier and variable. And, as undergraduates learn in their elementary logic course, 'The present king of France is bald' can be translated as $(\exists x)(Kx \; \& \; (\forall y)(Ky \supset x{=}y) \; \& \; Bx)$, where '$Bx$' means that x is bald and 'Kx' means that x is currently a king of France. (In words: There is currently a bald king of France such that every king of France is equal to him.) Once again, the denoting phrase has disappeared in the translation, to be replaced with quantified variables.

2.4 Logicism

What was significant about this method of translation was that it showed how the grammatical form of a sentence might differ from the logical form of the proposition the sentence expresses. Thus in the standard example, 'The present king of France is bald', the sentence has a subject, 'The present king of France', which does not correspond to any object in the proposition it expresses. The theory thus avoids the need to appeal to a shadowy realm of non-existent objects – often called 'Meinongian' although this is unfair to Meinong (see Oliver 1999) – to explain the meaning of the sentence.

This is a general method of considerable power. Wherever in philosophy we come across linguistic items which appear to refer to entities which are in some way problematic, the possibility now arises that the terms in question may be what Russell soon called 'incomplete symbols', that is to say expressions which have no meaning on their own but which are such that any sentence in which the expression occurs can be translated into another in which it does not. By this means we eliminate reference to the problematic entities without rendering meaningless the sentences which apparently refer to them.

The first application Russell made of this idea was to the case which had originally prompted him to examine the problem of the present king of France, namely that of classes. In *Principia Mathematica* (1910-13), written jointly with Whitehead, he developed a theory in which terms apparently referring to classes are incomplete symbols which disappear on analysis. The solution to the paradox Russell had discovered was to be that any sentence in which the term 'the class of all classes which do not belong to themselves' occurs would resist rewriting according to the translation rules and would therefore turn out not to express a proposition at all.

This solution does not just drop out all by itself, however. It is easy enough to formulate rewriting rules for eliminating class terms (so that, for instance, a proposition that appears to be about the class of all men turns out really to be about the property of manhood), but if that is all we do, we simply transfer the focus of attention to the corresponding paradox for properties (in Russell's terminology, propositional functions), which involves the property which holds of just those properties which do not hold of themselves. In order to avoid such paradoxes as this, Russell found it necessary to stratify propositional functions into types. Russell's theory is said to be 'ramified' because it stratifies propositional functions in two ways, once according to the types of the free variables they contain and then again according to the types of the bound variables.

Whitehead and Russell's aim in *Principia Mathematica* was an extension of Frege's. They wanted to embed not just arithmetic but the whole of mathematics in logic. If they had succeeded, they would perhaps not quite have solved the epistemological problem of how we come to know mathematical truths, but they would at least have made it subsidiary to the corresponding problem for logic. However, they did not succeed. Their principal difficulty was that the paradox avoidance measures they had to take do too much. In order to embed traditional mathematics in the theory of classes, we need to be able to count as legitimate many class terms that are *impredicative*, which is to say that the properties which define them make reference to the classes themselves and are thus ineliminably circular. In order that such class terms should count as legitimate it was necessary to assume the axiom of reducibility, which asserts that every such circular propositional function can be replaced by a logically equivalent non-circular one. But if *Principia Mathematica* was to be taken as showing that mathematics is part of logic, Whitehead and Russell had to maintain not only that the axiom of reducibility is true but that it is a truth of logic. And the reasons they gave for thinking that it is were unconvincing. A further difficulty was that in order to derive higher mathematics they had to assume the axiom of infinity, which asserts that there are infinitely many objects. Since they did not share Dedekind's conception of thoughts as objects, they could not adopt his 'proof'of this axiom. Their view therefore seemed to make the truth of higher mathematics depend on an unverified physical hypothesis.

Because of these difficulties over the axioms of reducibility and infinity, therefore, Whitehead and Russell's attempted reduction of mathematics to logic is generally regarded as a failure. Far more influential in philosophy, however, was the method of logical analysis of which it was an instance. The aim of this method, in application to any sphere of discourse, is to find the true logical form of the propositions expressed in the discourse. In the background, no doubt, was the hope that this would in turn, because of the conception of a proposition as made up of the things it is about, reveal the entities acquaintance with which the discourse requires. It was thus an assumption of the process, which Russell most of the time scarcely thought worthy of argument, that there *is* in this sense a determinate epistemological base to the discourse. Russell (1911) called this attitude 'analytical realism'.

2.5 Sense data

What, on this view, is the ultimate subject matter of ordinary discourse about the physical world? To answer this question we need to examine how Russell dealt with non-referring expressions. Russell analysed 'The present king of France does not exist' as $\sim(\exists x)(Kx \ \& \ (\forall y)(Ky \supset x{=}y))$. (In words: it is not the case that there is exactly one present king of France.) And an analysis of the same form is to be used in any case where we say that something does not exist. Thus, for instance, if we say that Homer did not exist, we should be taken to mean that no one person wrote both the *Odyssey* and the *Iliad.* Thus, Russell thought, we avoid the difficulties involved in supposing there to be a person, Homer, with the awkward property of non-existence. 'Homer' is thus for Russell an example of a term that is gramatically a proper name, but not logically so, since the correct logical analysis of 'Homer does not exist' reveals 'Homer' to be really a definite description in disguise. And in the same sort of way 'Sherlock Holmes does not exist' might be analysed by replacing 'Sherlock Holmes' with a definite description such as 'the detective who lived at 221b Baker Street'.

Russell used the term 'logically proper name' for any proper name which functions as such not just gramatically but logically – for any name, that is to say, which logical analysis does not reveal to be really a disguised definite description. But in ordinary language logically proper names are the exception rather than the rule. For it is not just words for spurious classical poets and fictional detectives that turn out to be disguised descriptions. The eliminative doctrine applies in any case where I can say intelligibly, even if falsely, that someone does not exist: since I can wonder whether Plato existed, 'Plato' is (at least in my idiolect) a disguised definite description. The same will apply to anything whatever of whose existence I can coherently entertain a doubt: the term referring to it must on this view be a disguised definite description.

It follows that a term '*a*' in my language can be a logically proper name only if the sentence '*a* does not exist' is not merely false but unintelligible: the object *a* must be something of whose existence I am so certain that I cannot intelligibly doubt it. This is a very demanding criterion: even tables, chairs and pens do not fulfil it since they might be holograms, tricks of the light or hallucinations. The only things in the physical realm that do fulfil the criterion, according to Russell, are sense data. Even if the green table on the other side of the room were an illusion, the patch of green at the centre of my visual field when I (as I think) look at it would certainly exist. It follows that if I say something about the table (that it is oblong, for example), the proposition that I express does not contain the table itself but may contain various sense data that I have experienced, such as the green patch just mentioned.

Where does this leave the table? At first Russell was inclined to infer its existence as the best explanation for the sense data. (If I look away or leave the room and come back in, the various sense data I experience have a regularity which is best explained by positing a table which causes them.) But later Russell was less inclined to ascribe any independent existence to the table and preferred to regard it as *constructed* out of the sense data. 'Whenever possible, substitute constructions out of known entities for inferences to unknown entities.' (Russell 1924)

By taking items of experience as building blocks in this way Russell showed evident sympathy with a central strand of empiricism, but he was very far from being a classical empiricist in Locke's mould, since he certainly did not think that they are the only constituents of propositions. He maintained a liberal ontology of universals such as love or meeting, which he thought were constituents of propositions such as 'John met Mary and fell in love with her'. Universals, he somewhat over-exuberantly claimed, are 'unchangeable, rigid, exact, delightful to the mathematician, the logician, the builder of metaphysical systems, and all who love perfection more than life.' (Russell 1912)

2.6 Difficulties with the theory

One curious side effect of Russell's theory is that it forced him to abandon the notion that modalities of possibility and necessity may be applied to propositions. The reason is as follows. Recall Russell's argument for the identification of the simple entities as those things whose existence it would be incoherent to doubt. The argument was that if *a* is a simple entity then the sentence 'I doubt whether *a* exists' cannot be intelligible, since if it is intelligible, the Russellian analysis will reveal '*a*' to be not a logically proper name but a disguised description, in which case *a* is not simple. We concluded, therefore, that simples are things whose existence is indubitable. But we can evidently run an exactly analogous argument in the case of the sentence 'it is possible that *a* does not exist': if this is intelligible, the Russellian analysis will reveal '*a*' to be a disguised description once more.

But if we simply use the second argument to place a further constraint on the simples, the theory collapses, since we now need the simples to be entities whose existence is not only indubitable but necessary, and even sense data do not fulfil this criterion: I may be sure that there is a patch of green in the centre of my visual field, but can I not also represent to myself the possibility that it might not have been (if, for instance, I had painted the wall a different colour)?

The only way out for Russell if there are to be any simples in the world at all is to say that despite appearances to the contrary I cannot in fact represent the possibility of there not having been that sense datum. If talking of propositions as possible is to be legitimate, it will have to be explained as a way of saying something not about how the world could have been but about how it actually is. If I say that I could have been killed cycling to work this morning, for instance, I am really saying something about how busy the traffic was on the main road or how carelessly I was steering.

Frege, we have seen, made explaining communication one of the central tasks of his theory of meaning: that is why he had to insist that the sense of an expression is not simply an idea in my mind but a distinct, inter-subjectively available entity. For Russell, on the other hand, it was not really part of the task he was engaged in to explain communication: on his view the fact that we communicate at all emerges as a strange kind of miracle. For the sense data experienced by me are not the same as those experienced by anyone else. Even if you are in the room with me, the angle at which you look at the table, and hence the exact sense data you obtain from it, will be different. As a consequence the logically proper names in my language do not mean the same as those in yours (see Russell 1918, §II). The only entities the propositions you and I express have in common are universals. Since the propositions of mathematics and logic, Russell thought, have no components that are not universals, there is the prospect that we can genuinely communicate them, but in all other cases some degree of failure seems inevitable.

Russell's theory is thus at risk of a kind of solipsism. At first sight it might also be thought to flirt with idealism. The sense datum I experience is private in the sense that no one else but me has experienced it. It seems a short step from there to the claim that the sense datum is an idea in my mind. But if we say that, then the world is constructed out of ideas, and this is idealism.

So at any rate a casual reader of *The Problems of Philosophy* might think. But it is nor Russell's view (or Moore's). Something is not a sense datum unless it is experienced, but saying that does not commit us to identifying the sense datum with the experience. Russell and Moore both conceived of sense data as objective entities to which we may bear a relation of acquaintance (Russell) or direct apprehension (Moore). Sense data may, they thought, exist when they are not being experienced; and among the things of the same sort as sense data – Russell called them 'sensibilia' – there may be some that no-one ever has experienced or ever will experience. To say that no sensibile is a sense datum unless someone is sensing it is thus on their view much like saying that no man is a husband unless there is someone he is married to.

2.7 The multiple relation theory of judgment

A proposition, according to Russell and Moore, is a sort of complex made up out of entities of various sorts – sensibilia, ideas, or universals. If I give two sense data I am experiencing the names '*a*' and '*b*', for instance, the sentence '*a* is above *b*' might express a proposition which consists of *a*, *b* and a certain spatial relation (a universal) of aboveness. But what it is for *a* actually to be above *b* is just that there should be a complex consisting of *a*, *b* and this spatial relation. The proposition may be thought of as asserting the existence of a certain fact. So in the case where the proposition is true, it is identical with the fact whose existence it asserts. But what of the case where the proposition is false? In that case there is no fact, as there is when the proposition is true. It is hard to see how there can be a complex consisting of *a*, *b* and aboveness if *a* is not in fact above *b*, since what it would be for *a* to be above *b* is just that there should be such a complex.

The solution to this problem, Russell came to think, was to eliminate propositions from the account of what it is to make a judgement. And Russell's logical method apparently gave him the means to achieve this. In '*A* judges that *p*', the expression apparently referring to a proposition *p* was to be treated as an incomplete symbol to be eliminated on analysis, in much the same manner as the present king of France, so that the judgement would turn out to consist not in a binary relation between the person *A* who makes the judgement and the proposition that is judged, but in a multiple relation between *A* and the various components of the erstwhile proposition. So, for instance, 'I judge that *a* is above *b*' will turn out on analysis to express a relationship between four entities: me, *a*, *b* and aboveness.

Now one might think that this theory is at risk of a regress: it eliminates the proposition *p* from the analysis of '*A* judges that *p*', to be sure, but is not '*A* judges that *p*' itself another proposition requiring analysis in turn? Presumably, though, Russell was proposing an analysis not of the proposition '*A* judges that *p*', but only of the judgment itself, i.e. of the fact (when it is fact) that *A* judges that *p*. Since the difficulty that led him to adopt the theory was only a difficulty with false propositions and not with facts, there is no problematic regress at this point.

There is, however, a different problem. Notice that the judgement relation is according to this theory not only of multiple but of varying adicity: in the case just mentioned it is a quaternary relation, but that is only because the proposition being analysed has three components; other cases would be different. Not only that but there is no constancy about which elements of the judgement should be of which kinds (sense data, universals, or whatever). As a consequence the judgement relation has to be supposed to be very tolerant as to what sorts of arguments it takes. In its first position, of course, we may suppose that it always has the person *A* making the judgement, but in its other positions it will have to tolerate all sorts of combinations of entities. So it is hard to see how the form of the relation can be such as to determine whether the various entities in these positions are such that it even makes sense to suppose that *A* makes a judgement concerning them. The theory makes it seem, for instance, as if one could judge that the table penholders the book.

Wittgenstein, who was at that time still officially Russell's student at Cambridge, pointed out this difficulty to him in the summer of 1913. 'Every right theory of judgment,' Wittgenstein said, 'must make it impossible for me to judge that this table penholders the book. Russell's theory does not satisfy this requirement.' (Wittgenstein 1913, 3rd MS) Moreover, since the objection depends not on detailed features of Russell's theory but only on its overall shape, it is presumably devastating. At any rate it devastated Russell, who abandoned forthwith a book he was writing (*Theory of Knowledge*) in which the theory played a central role.

3 The *Tractatus*

3.1 Propositions

But if Wittgenstein had disposed of Russell's theory, he had not disposed of the need which it was intended to fill. What was needed, he repeatedly urged, was 'a correct theory of propositions'. The problem of false propositions which Russell tried to solve by means of the multiple relation theory had arisen from Russell's conception of propositions as complexes. He had started, that is to say, from the view that 'The book is on the table' and 'the book' both refer to complex entities, and had tried to analyse these entities in similar ways. Wittgenstein's starting point was the realization that there is a fundamental error in Russell's way of conceiving the matter. Sentences are not like names, and the reason they are not like names lies precisely in the feature which had led to Russell's puzzlement, namely that they are capable of truth and falsity.

Wittgenstein called this the bipolarity of the proposition. He was especially struck by the symmetry that exists between a proposition and its negation, a symmetry which Russell's conception of propositions as complexes did not account for. Wittgenstein thought of p and $\sim p$ as being two sides of the same coin, and hence rid himself of the temptation to think of one of them as essentially more complex than the other. There is no more reason to think that negation is in some way a constituent of $\sim p$ than that it is a constituent of p, and hence no reason to think that it is a constituent of either. 'My fundamental thought,' he said, 'is that the "logical constants" do not represent.' (1922, 4.0312)

How, then, is the bipolarity of the proposition achieved? Wittgenstein's answer to this question is famously known as the picture theory: a proposition pictures how the world would have to be for the proposition to be true; the proposition is true if things are as it pictures them to be. Wittgenstein's theory avoids Russell's difficulty over false propositions because the entities which make up the proposition are not the real-world objects but only linguistic proxies for them – names. Wittgenstein's was nonetheless an identity theory, as Moore's had been, and not a correspondence theory. For the names are arranged in the proposition (picture) in the *same* way as their real-world correlates, the objects, are arranged if the proposition is true. The theory thus nicely sidesteps Frege's powerful objection to correspondence theories, that correspondences come in degrees but truth does not. 'What is only half true is untrue,' as Frege (1918, p. 60) succinctly puts it.

So far, though, picturing is only a vague metaphor. Plainly much more would have to be said if we wanted it to amount to a semantic theory, and it is far from clear, to me at least, whether it can be said in such a way as to make the theory coherent. Lying behind the picture theory, however, there is what seems to me to be a genuine insight, of which there are glimmerings in Frege, but which Wittgenstein was the first to bring fully to light: it is an essential component of what enables a sentence to express something about the world that the complexity of the proposition the sentence expresses should track the complexity of the possibilities of arrangement of the world which it represents.

Wittgenstein's way of cashing out this insight is to conceive of a proposition not merely as saying how the world is but as contrasting how it says the world is with other ways the world could have been but isn't. The role of a proposition, we might say, is to divide all the possible worlds into two classes: if the actual world is in one class, the proposition is true; if it is in the other, the proposition is false. The bipolarity noted earlier is explained by the fact that the negation of the proposition divides the world into exactly the same two classes: what is reversed is only which class is to count as true and which false. (Wittgenstein calls the division of possible worlds which a proposition effects its *sense*.) A *tautology* is a proposition which is true in all possible worlds; dually, a *contradiction* is one which is true in none. Wittgenstein called these two extreme cases *senseless* because, placing all the possible worlds in one class or the other, they cannot really be said to divide them at all.

Notice, then, that the notion of possibility is built into the expressive nature of propositions from the start. This fact makes vivid how different Wittgenstein's conception was from Russell's. Russell's conception had forced him to abandon the notion that propositions may be possible or necessary at all, whereas for Wittgenstein it is precisely this that makes them expressive. For Russell an entity can be simple only if its existence is indubitable, whereas for Wittgenstein the simple entities (which in the Tractatus are just called 'objects') are just those that are necessarily existent. The role of propositions, on Wittgenstein's view, is to express possible configurations of the world; the role of objects is to be the hinges around which these possibilities turn. What varies between possible worlds, that is to say, is not what objects there are but only how they are combined with one another to form states of affairs. What makes language expressive is that the substitutional possibilities of the linguistic elements which it allows for match precisely – are identical with – the combinatorial possibilities of the objects these elements represent. That 'John' and 'Adam' are words of the same grammatical category is the linguistic correlate of the fact that John and Adam themselves are capable of getting into just the same situations.

3.2 Mathematics

We saw earlier that Wittgenstein's conception of the sense of propositions gave him an elegant criterion of logical truth. Using this criterion Wittgenstein showed that Russell's axiom of reducibility is not a logical truth. So much the worse, Wittgenstein thought, for mathematics. Rather than try to repair Russell's system so that mathematics would consist of tautologies, he simply ditched it, or most of it: the only part of mathematics he kept was simple arithmetic, equations such as 7+5=12. Equations, he held, do not express genuine senseful propositions, but nor are they logical truths (i.e. tautologies). Instead, they have the same form as general claims that certain sorts of symbols express tautologies.

We need not go into the details of Wittgenstein's account of mathematics here (see Potter 2000, ch. 6). What is important here is to note that Wittgenstein opposed the idea that mathematics consists of tautologies, and yet went out of his way to emphasize in the *Tractatus* how *similar* the equations of mathematics are to tautologies:

The logic of the world, which the propositions of logic show in tautologies, mathematics shows in equations. (Wittgenstein 1922, 6.22)

If this is the similarity, then, what is the difference? The fundamental difference between tautologies and equations lies in how they can be applied. A tautology, such as $p \vee \sim p$, can be seen as a sort of limiting case of a genuine proposition. (For more on this see §3.3 below.) Its component parts, such as *p*, have sense, and the ways in which those parts are combined to form the whole are ways in which propositions with sense can be formed. It is just that in this case the sense so formed turns out to be empty. What happens when we try to form a parallel explanation of the equation 7+5=12? The intended application is that this equation allows us to infer such facts as that if there are 7 apples and 5 oranges then there are 12 pieces of fruit. The general principle the equation encodes is thus:

(A) If the number of *F*s is 7, the number of *G*s is 5 and nothing is both an *F* and a *G*, then the number of things that are either *F*s or *G*s is 12.

But this is now plainly not parallel to the tautology case. For no one instance of (A) carries the import of the equation 7+5=12. If we try to treat the equation as meaning the universal generalization of (A), we run into a technical difficulty connected with the theory of types, namely that we can only generalize over one level in the hierarchy at a time, which is not what we want: we ought to be able to count first-level properties by just the same means as we count apples. But even if we prescind from this difficulty and focus only on the case where what we are trying to count are Wittgensteinian objects, we still do not get what we want: it is possible that no first-level property has just five instances, and in that case the equation 7+5=13, interpreted according to the current proposal, would come out true, as would every other equation with the number 5 on the left hand side (because the antecedent of the conditional would be uninstantiated). This sort of accidental truth is plainly not what we were aiming for, so the only thing left to us is to interpret 7+5=12 as meaning that (A) is not merely always *true* but always *tautological*. This, though, cannot *itself* be a tautology since it is at the wrong semantic level for that: as we are about to see, nothing which expresses that something is a tautology is, according to the *Tractatus*, itself a tautology.

3.3 Saying and showing

We saw earlier how Russell's atomism was threatened by the incompatibility of the twin constraints that the simple entities should be both indubitable and necessary. His solution was to abandon the notion of possibility. Wittgenstein was threatened by the same incompatibility, but he could not abandon the notion of possibility, since it was central to his conception of the expressibility of the proposition, and therefore abandoned propositional attitudes instead. In other words, he was led to the view that '*A* believes that *p*' and '*A* doubts whether *p*' and their ilk are not propositions. In particular, 'I doubt whether *a* exists' is not susceptible to the standard Russellian analysis, so we have no reason to suppose that the simples are things whose existence we cannot doubt, and no reason, therefore, to identify them with sense data.

If '*A* believes that *p*' is not a proposition, what is it? Wittgenstein's gnomic utterance tells us only that it is of the same form as ' "*p*" says that *p*'. (1922, 5.542) His idea was that for *A* to believe that *p* is for *A* to have in mind a symbol of an appropriate sort which says that *p*. The key element in the holding of a belief is thus the ascription of sense to a certain symbol. But this ascription is *not* a proposition. More generally, nothing can be a proposition that attempts to express the expressiveness of a symbol. So, for instance, we cannot say that the name '*a*' refers to *a*.

To see why Wittgenstein made this claim, we need to contrast it carefully with another that is superficially similar. Wittgenstein distinguished between a sign, which is an arrangement of words (or, in the degenerate case, a single word), and a symbol, which is what the sign becomes when I read it as saying something. That the *sign* 'Snow is white' says that snow is white is plainly a contingent fact about English: the word 'white' might have meant black, for instance. But anyone who is fluent in English will, on seeing the sentence, immediately read it as saying what it says in English: they will, as Wittgenstein would put it, see in the sign a particular symbol. And it is not contingent that that *symbol* says that snow is white: if it said something else, it simply wouldn't be the same symbol.

This shows readily enough, I think, that

The symbol '*p*' says that *p*

is not a proposition with sense, i.e. something that is true in some possible worlds and false in others. And in the same way we can understand why

The name '*a*' refers to *a*

is not a proposition with sense either.

What is harder to see is why they cannot be tautologies. To see this let us compare them with

Either it is raining or it isn't.

This does not express a sense: it does not, in Wittgenstein's terms, divide the possible worlds into two classes. But this is only because it puts all the worlds into one class: it has the right general shape to be a proposition with sense, but its parts cancel one another out and end up saying nothing. We can see this by noting that we can approximate what it says (i.e. nothing) more and more closely by propositions that do have sense. For instance:

Either it is raining or it is snowing
Either it is raining or it is snowing or it is overcast
Either it is raining or it is snowing or it is overcast or it is sunny
…

If we carry on like this, eventually we list all the ways the weather could be, and the resulting disjunction says nothing about the weather at all, i.e. it is a tautology.

But return now to our earlier example, 'The name "*a*" refers to *a*.' It is not hard to convince oneself that there is nothing analogous we can do to approximate this by means of propositions with sense. Hence we are forced to conclude that it is not senseless but *nonsense*: it is not something of the right shape to have a sense which ends up cancelling out and saying nothing – not a limiting case of senseful propositions – as 'Either it is raining or it isn't' was; rather is it something which is not of the right shape to have a sense at all.

The examples of nonsense which we have considered so far are what might broadly be called semantic: they attempt to say what it is that some symbol expresses. But once Wittgenstein had identified the category, there were many other sorts of discourse that he decided should be put in it. Consider, for example, an ethical claim such as 'Killing babies is wrong'. It is easy to see that this is not a tautology: not only is it fairly obviously not a matter of logic, but it does not have the sort of triviality that 'Either it is raining or it isn't' has: it cannot be approximated in the same manner by senseful propositions. What is harder in this case, in contrast to the semantic examples considered earlier, is to see why it is not a contingent truth. But if it were contingent, there would be some possible worlds where killing babies is wrong and others where it is not. What if the actual world happened to be one of those in which killing babies is not wrong? One might be tempted to say then that that would be a worse *world* than one in which killing babies is wrong. But if one said that, then it would really be this last claim that was doing the ethical work, not the original claim that killing babies is wrong. Either way, therefore, the claim which carries the ethical content is not a contingent truth. Since there are in Wittgenstein's system only three categories – senseful, senseless and nonsensical – we must conclude that sentences making ethical claims are nonsense.

The same goes for almost all the spheres of discourse which philosophy has traditionally found problematic – aesthetics, religion, scientific laws, the relationship between mind and body. In all these cases, and others, Wittgenstein held that the solution to our philosophical difficulties is, properly speaking, their *dissolution*. Our mistake was to treat as senseful propositions linguistic expressions which turn out to be nonsense.

3.4 Important nonsense?

What is most important here is to see what the scope of Wittgenstein's argument for nonsense is. Notice, in particular, that the argument does not depend on some of the features of Wittgenstein's system that have subsequently been rejected, such as his atomism or his assumption that elementary propositions are independent. Notice, too, that it cannot simply be assimilated to arguments such as the liar paradox which depend on diagonalization arguments. Indeed the conclusions of these diagonalization arguments are typically weaker than Wittgenstein's because they demonstrate only the *relative* inexpressibility of the notions in question (in the case of the liar paradox, truth). The liar paradox shows, that is to say, only that the truth predicate for a language cannot consistently belong to the language itself. But the now familiar Tarskian resolution of the paradox simply recognizes a hierarchy of languages: the notion of truth for any language in this hierarchy is expressible in the next language up.

There is of course nothing remotely surprising about the fact that for each language there are notions which that language cannot express. (Classical Latin has no word for a mobile phone, for example.) All that was ever surprising about the liar paradox was that truth turned out to be such a notion. The inexpressibility which Wittgenstein demonstrates, on the other hand, is of a radically different kind, since what he shows is that what we are trying to say simply does not have the right shape to be said in *any* language, however extended, provided only that the language obeys the fundamental Tractarian constraint that it aims to distinguish between ways the world could be. So in any case of Tractarian inexpressibility moving to a meta-language will not do the trick.

Part of what is powerful about Wittgenstein's inexpressibility argument, then, is its generality. But notice also, on the other hand, how restricted its conclusion is. 'Nonsense' in the *Tractatus* is, as we have seen, a technical term defined in contrast to 'sense'. Even if we accept the Tractarian picture according to which the *primary* purpose of any functioning language is the expression of sense, it does not follow that that is its only purpose: we would need a further argument if we wanted to conclude that any linguistic item which does not succeed in expressing sense is simply gibberish. Not only does the *Tractatus* not supply such an argument; it is plain that Wittgenstein himself did not believe the conclusion. There is ample testimony to the importance he placed on ethics and religion (not only then but throughout his later life).

Even if we ignore ethics and religion, moreover, it would be hard to hold resolutely to the view that in the *Tractatus* all nonsense is gibberish, given that what is there characterized as nonsense includes not only such mundane items as ascriptions of belief but also arithmetical equations such as 7+5=12.

There is a danger, therefore, that the emphasis recent work has placed on a contrast between so-called 'old' and 'new' (or irresolute and resolute) readings of the *Tractatus* (see, for example, McCarthy and Stidd 2001) may create a polarized debate between two equally implausible extremes. If the old, irresolute reader is supposed to be someone who thinks that nonsense can be appropriately expressed by moving to a metalanguage, then it is hard to find a respected commentator on the book who counts as irresolute. (The nearest, perhaps, is Russell, who briefly canvassed the idea in his introduction to the *Tractatus*, but even he immediately noted that this was not Wittgenstein's own view.) And nonsense, on the other hand, is no doubt nonsense; but a resolute reader who steadfastly maintains that nonsense is simple gibberish misses the subtlety of Wittgenstein's view.

The challenge to all readers of the *Tractatus*, whether they choose to label themselves new or old, is to explore the constraints there plainly are on which nonsense we may utter in which circumstances – constraints which do not apply to gibberish. The *Tractatus* offers us a reason why logic does not apply to nonsense, a reason whose attraction is that it contrives in a recognisable sense not to threaten the universality of logic. That, if it is right, is an important conclusion. It is equally striking, however, how much nonsensical sentences have in common grammatically with senseful ones. The same observation, of course, could be made about Lewis Carroll's nonsense verse, 'Twas brilling, and the slithy toves / Did gyre and gimble in the wabe.' Much more would have to be said, however, before we could derive from the *Tractatus* the suggestion that the psychological effects of the sentence '7+5=12' are importantly analogous to those of nonsense verse, or that the constraints on correct mathematics are anything like those on good poetry.

One does well to remember that when Wittgenstein said that he believed himself to have found the final solution to the problems of philosophy, he meant what he said. In particular, he intended the doctrine of saying and showing to solve (or more properly, once again, to dissolve) the problem of the relationship between the self and the world – the problem, that is to say, to which realism and idealism represent contrasting solutions. His thought was that the things which cannot be said but only shown – symbolic expressiveness, ethics, aesthetics, God – are all different aspects of this relationship. And their absolute unsayability was for him a way of coming to see that what this is is not really a *relationship* at all. The traditional philosophical picture, let us recall, sees a problematic gap between the self and the world, which realism attempts to bridge. Idealism obviates the need for a bridge by removing the world from the picture. What Wittgenstein does, by contrast, is in a certain sense to remove the self. Or, more accurately, he conceives of my self as constituted by the process of representing the world in which I am engaged. And what we are showing when we speak nonsense is always an aspect of this process.

If this is right, then the consequences for philosophy are far-reaching indeed. All the 'big' questions of philosophy are according to the *Tractatus* not really questions at all and cannot be answered by the application of logical reasoning in anything like the manner that Russell and others were attempting. For logical reasoning applies only to propositions, and the sentences which occur in 'big' philosophy do not express propositions. Wittgenstein's closing admonition, 'Whereof one cannot speak, thereof one must be silent' (1922, 7), therefore enjoins us not to try to discuss these questions. It certainly does not follow, however, that we should dismiss them as worthless. There will be other processes – of more or less self-conscious reflection, perhaps, or of prayer – which may lead us to awareness that killing babies is wrong, that a painting is beautiful, or that God exists.

3.5 Reactions to the Tractatus

There is certainly something very mystical about Wittgenstein's view of the unsayable, and it is unsurprising that neither Cambridge atheists such as Russell nor scientistic positivists in Vienna such as Carnap were inclined to take much notice of it. What they took much more seriously at first was Wittgenstein's dismissal of the logicist reduction of mathematics to the theory of classes. He rejected it because, as he put it,

> The theory of classes is altogether superfluous in mathematics. This is connected with the fact that the generality which we need in mathematics is not the *accidental* one. (Wittgenstein 1922, 6.031)

According to Wittgenstein, let us recall, mathematical theorems (to the extent that he granted them house room at all) are not themselves tautologies but have the form of claims that various other symbols are tautologies. There seemed to be little prospect of giving an account of any more than elementary arithmetic in accordance with this view.

What was needed if Wittgenstein's view was to be refuted, therefore, was a demonstration that the theorems of the theory of classes were indeed simply more elaborate tautologies. The person who attempted this was Ramsey, in his paper on 'The foundations of mathematics' (1926). What he argued was that the theory of classes could indeed be regarded as part of logic because of a logical notion that he called a 'propositional function in extension'. On Wittgenstein's understanding, a propositional function is what we obtain if we take a proposition and replace some symbolic element in it with a variable. Thus, for instance, the proposition 'Socrates is dead' gives rise to the propositional function '*x* is dead'. If we now replace the variable in this propositional function with another name, 'Plato' for example, we obtain the proposition 'Plato is dead', which in an immediately recognisable sense says the same thing about Plato as the previous proposition did about Socrates. Ramsey's new notion, by contrast, is simply a function (in the mathematical sense) taking objects to propositions: we might, for instance, define a propositional function in extension φ so that

$$\varphi(\text{Socrates}) = \text{Queen Anne is dead}$$
$$\varphi(\text{Plato}) = \text{Einstein is a great man.}$$

The difficulty Ramsey's notion was designed to overcome is that if we combine Wittgenstein's understanding, according to which φa must say the same about a as φb says about b, with Whitehead and Russell's idea that talk about classes is to be reduced to talk about propositional functions, we obtain the result that the only sort of class we can talk about is, in Wittgenstein's terminology, *accidental*, i.e. a class of things having some property in common. We cannot talk about the *essential* classes which we need in mathematics, e.g. classes defined by enumeration such as $\{a,b\}$. Another usage would be to call the first notion *de dicto* and the second *de re*, since they differ in how they vary across possible worlds. In a world in which a and b happen to hold all their properties in common, the *de dicto* notion is unable to retrieve the *de re* class $\{a,b\}$. With Ramsey's notion, by contrast, we can talk about the class {a,b} by defining a propositional function in extension which expresses tautology if $x=a$ or $x=b$ and expresses contradiction otherwise.

If Ramsey's notion of propositional function in extension were indeed, as he claimed, an 'intelligible notation', we would therefore be well on the way to resurrecting Russell's logicist programme. Unfortunately, however, it is not. If we wish to claim that mathematics consists of tautologies, it is no use treating equations as merely abbreviated embodiments of their intended applications: the only course is to treat them as tautologies in their own right, their tautologousness not being seen as derived from their applications. Ramsey's account is evidently an instance of this general strategy. But if we do this, we shall eventually have to explain how these tautologies nevertheless do get applied: we shall have to establish a connection between the new ways of expressing senses thus introduced and the old ones. But now our difficulty is that we have broken the link with a crucial aspect of Wittgenstein's account of tautologies described earlier, namely that they can be seen as limiting cases of genuine propositions, i.e. as trivial cases of forms capable of expressing non-trivial senses. Without that link mathematics floats free of the rest of language and the account lapses into a version of formalism.

That is the philosophical reason for Ramsey's failure: for the details consult Potter (2000, ch. 8). There is also a technical reason which was discovered by Gödel just after Ramsey's premature death in 1930. One way of expressing what Gödel's incompleteness theorems demonstrate is that arithmetic (and mathematics more generally) have a complexity that tautologies do not have. This shows that mathematics cannot simply be regarded as consisting of more complicated tautologies: the difference is one of kind, not degree. The incompleteness theorems, in other words, force us to recognize a distinctively mathematical notion of necessity distinct from the logical notion of tautology picked out by Wittgenstein.

The claim that there is only one kind of necessity, namely logical necessity, was in fact the first of the Tractarian doctrines that Wittgenstein himself retracted, but his reason was not mathematical. In the *Tractatus* he had admitted it as necessary that nothing is both red and green simultaneously. Since he then held that the only sort of necessity is logical necessity, he was forced to conclude that red and green are not simples but have some analysis from which the incompatibility emerges as tautological. But he did not trouble to supply the required analysis, or even sketch how it might go.

When he resumed philosophy in the late 1920s, he began to meet members of the Vienna Circle such as Schlick, Waismann and (for a time) Carnap. Their approach to philosophy, heavily influenced as it was by scientific method, was certainly not to Wittgenstein's taste. Nonetheless, it may well have been their influence that led him to wonder how the analysis of colour words is actually supposed to proceed. Moreover, it is not just colour incompatibilities that have to be dealt with. If I look at a lamp, a patch in my visual field is filled with light of a certain intensity: the same patch cannot simultaneously be filled with light of another intensity. This incompatibility, too, would according to the *Tractatus* have to be analysable in some way. When Wittgenstein came to realize that it cannot (Wittgenstein 1929), he abandoned the doctrine that elementary propositions are independent. In other words, he came to hold that there are internal relations – necessary relationships – between atomic facts.

This is not perhaps such a major retraction: in the *Tractatus* he simply asserted the doctrine of the logical independence of atomic facts without argument, and one might even wonder whether he had simply taken it over from Moore. More significant, however, is the problem of identifying the simple entities which logical atomism presupposes. We have seen that Russell took them to be sense data. Wittgenstein did not, but made only one remark in the *Tractatus* about what else they might be: 'Space, time and colour (colouredness) are forms of objects.' (1922, 2.0251) Points in space, moments in time[4] and colouredness (but not, as we have just seen, the various colours such as red and green) are therefore Tractarian objects. It might be thought surprising that he said so little about such an apparently central question, but in a way what is more surprising is that he said even this much. For there is a sustained passage in his wartime notebooks (June 1915) in which he lays out the difficulties there are in supposing that we have *any* stable conception of what is simple in the world.

What he evidently recognized in 1915, but chose in the finished book simply to ignore, was that what we take to be simple is highly sensitive to context, shifting not just from one conversation to another, but even from sentence to sentence. In his later philosophy Wittgenstein tried to capture something of this sensitivity to context by means of the notion of a 'language game': our language is to be thought of not, as in the *Tractatus*, as a single unit, but as an overlapping patchwork of sub-languages (games) in which different (and sometimes conflicting) notions of simplicity may be at work.

It is a truism of modern Wittgenstein scholarship that the *Philosophical Investigations* do not represent the clean break from the *Tractatus* that was once supposed: the similarities between early and late are as significant as the differences. One example of this is the continuing importance in his later work of the idea that the expressiveness of a proposition is inherently contrastive, so that something can make sense only if its negation also makes sense. It is, for example, a repeatedly exploited thought in his later work that in order for us to count something we do as correct we must have an account of what it would be for it to be a mistake.

This continuity in thought between early and late is especially apparent in the notion of the unsayable. The perplexity which the later Wittgenstein encourages in us about what it is to follow a rule cannot be dissolved by means of a further rule, since the new rule would merely inherit the same perplexity. Yet Wittgenstein does not intend our perplexity to be permanent: we do indeed apply rules correctly all the time. When he invokes what he calls 'our form of life' as a solution to the problem, he intends it to play much the same role as the metaphysical subject plays in the *Tractatus*. The point of the rule-following considerations is to free us of a conception he takes to be misleading – the conception, that is to say, according to which there can be any further question as to whether our application of the rule is *really* correct if we take it to be so. And this is just the same picture whose abandonment Wittgenstein recommended in the *Tractatus* as a way of dissolving the dispute between realism and idealism.

[4] Or perhaps regions of space and intervals of time – Wittgenstein does not say which.

This has been a recurring theme in 20th century philosophy, taken up with considerable sensitivity by Putnam (1981), for example. His use of the so-called permutation argument has much in common with Wittgenstein's use of the rule-following considerations: he aims not to question whether 'cat' really refers to cats but to reject the idea, central to what he calls metaphysical realism, that there is a perspective from which we can coherently ask whether it does or not.

4 Analytic philosophy

4.1 What it is not

The survey we have given of themes in the birth of analytic philosophy is certainly selective, as is inevitable in a volume of this kind. Nonetheless, there would, I think, be widespread agreement that what I have described are at any rate *some* of the origins of analytic philosophy. The fact of this agreement is itself quite remarkable: not all intellectual movements have such clearly identifiable births, nor ones so localized. But when one tries to identify philosophical views that characterize analytic philosophy, the picture becomes murkier: it is surprisingly hard to find a coherent cluster of views that would be subscribed to by all those 20th century philosophers who have been taken to belong to the analytic tradition.

The idea which gave the tradition its name, that an analysis of sentences could reveal the true structure of the propositions they express and hence the true nature of the world, has re-emerged in various forms, and is not yet quite dead, but it certainly is not universally accepted. Followers of Quine, for example, have held that no sentence-by-sentence analysis can hope to explain what we are saying. The correct way to understand the relationship between language and the world was not even a point of agreement between the founders of the tradition, let alone their inheritors. And the assumption, prominent in philosophy since ancient times, that there is anything we might term the *given*, an unanalysable substance of which the world is composed, seems to be believed by hardly anyone in the analytic tradition nowadays.

One thing that analytic philosophers have certainly had in common has been a belief that natural science, as it has been practised in the last couple of centuries, has conformed very largely to the norms of rationality, and that its evident success owes much to its employment of these norms. That, however, is scarcely enough to distinguish analytic philosophers from anyone else. Many of them have also been tempted to argue in the other direction – to use the practice of natural science as an aid to identifying these norms, and its success as a justification for them. But it is a further step, on which they have certainly not all agreed, to claim that the norms exemplified in the practice of natural science are the only rational norms we have.

I mentioned earlier Frege's 'linguistic turn'. Part, at least, of what this involved was his realization that if we are to analyse the structure of thought, we have no choice but to engage in an analysis of language, for the straightforward reason that, except perhaps in the first-personal case, language is our primary means of access to thought. Whether it is also constitutive of the linguistic turn to claim that language is our only means of access is more controversial, however. This stronger claim has been repeatedly urged by Dummett, who has even asserted (Dummett 1993) that an acceptance of it is a necessary condition for anyone to count as an analytic philosopher.

It is of course unsurprising that a precise delineation of a hitherto vaguely understood boundary should place a couple of cases on unexpected sides of the fence, so Dummett is no doubt right to be unperturbed that Gareth Evans and Christopher Peacocke, for example, do not count according to his definition as analytic philosophers. But what about Russell? In the great phase of his work we have been discussing here (between 1898 and 1914) Russell always conceived of the subject matter of philosophy as consisting of abstract configurations of parts of the world. He changed his mind, as we have seen, about what these configurations are (facts, propositions, judgements); and it was a profound insight for him when his discovery of the theory of descriptions led him to the idea that the surface grammar of a sentence can mislead us significantly as to the structure of the part of reality to which it corresponds. But although this insight led him to be somewhat more careful than before about the distinct structure of language, it did not lead him to the linguistic turn in Dummett's sense. It was only after 1918 that Russell abandoned the view that logic is transparent (see Russell 1959, p. 145) and became interested in the relationship between language and fact.

Even in his later philosophy, however, Russell would still not count on Dummett's view as an analytic philosopher because taking up the study of meaning led him directly to abandon the form of anti-psychologism which Dummett takes to be another essential characteristic of analytic philosophy: Russell abandoned, that is to say, the view that 'the study of *thought* is to be sharply distinguished form the study of the psychological process of *thinking*.' (Dummett 1978, p. 458)

In Frege's hands anti-psychologism was a thesis about logic with normative content: logic is the study not of the laws by which we in fact think but of those by which we ought to think; and the normativity of the 'ought' here was not, Frege thought, simply to be resolved into an account of the benefits that accrue if we reason according to these rules rather than others.

This normativity is something Carnap explicitly renounced, at least for a time. 'In logic', he said in (1934), ' there are no morals,' because what counts as a logical truth depends on the linguistic framework we adopt and this choice is determined only by pragmatic, not normative, constraints. Carnap did not hold this ruthlessly pragmatic line for very long, but even while he did, he did not thereby rid logic wholly of normativity: it remained the case that once we have adopted a framework, what follows from what within the framework is a determinate matter that admits of right and wrong.

Something similar applies to the later Russell. During his most psychologistic phase, he thought that 'the non-mental world can theoretically be completely described without the use of … logical words' (Russell 1938, p. 43). Concepts like disjunction and negation are required, he thought, only because of 'such mental phenomena as doubt or hesitation'. He did not say, and it does not follow, however, that once we have acquired these concepts their properties are up to us to settle.

Most extreme of all was the later Wittgenstein, whose endeavours to expose what the normativity of logical reasoning amounts to led him to deconstruct it completely. Even in his case, however, the aim was to reject an inappropriate picture of normativity rather than to give us a licence, when arguing, to say whatever we like.

Another feature which has been offered as distinctive of analytic philosophy is what one might call the one-level view of language – the view, that is to say, that all cognitive content is factual content (see Skorupski 1997). The discussion of the Tractatus earlier will have made plain how little sympathy I have with this view or, therefore, with the idea that it might be essential for an analytic philosopher to hold it. It was no doubt an influential strand in logical positivism, and many Quineans seem to take naturalism to be somehow an endorsement of something very like it, but Wittgenstein did not share it, early or late, and it is not widely held today outside Quinean circles.

That the view was ever influential is indeed attributable to a failure by its proponents to appreciate the role of the metaphysical subject in Wittgenstein's philosophy. That my language is *mine* makes it normative in a way that a simple listing of its rules does not capture. The point is quite general: if we identify any process as constitutive of our rationality, we must recognize that a bare description of that process will inevitably fall short of representing what is involved, since it will leave out the further fact that the process is ours. The error that consists in failing to realize this is one that has been made not only by the positivists. It may be traced too, for example, in a kind of argument for physicalism that has found favour more recently. Even if modern physics were all that our best theory of the world came to, there would be a further fact, not contained in the theory itself, namely that it was indeed our best theory.

One might be tempted, therefore, to conclude that the analytic tradition is no more than that – a tradition; to conclude, that is to say, that what unites its practitioners is only that they agree on the historical origins from which their disparate approaches to philosophy have stemmed. This is no doubt helpful, but it is in the end too coarse-grained, not so much because there have been philosophers outside this historical tradition (such as Bolzano) whom we would nevertheless wish to describe as analytic, but rather because there have been many in the 20th century who took inspiration from the authors I have discussed here but who would generally be considered to lie outside this tradition.

Another method that is tempting is to define analytic philosophy by what it is not. And no doubt this too has its point. Just as Protestantism has, historically and to some extent theologically, been defined by its opposition to Roman Catholicism, analytic philosophy has undoubtedly acquired its identity partly by its oppositions, first to what is unhelpfully described as continental philosophy (Hegel, Nietzsche, Heidegger, Sartre) and then more recently to post-modernism. But these oppositions, although they tell us something about the nature of analytic philosophy, do not tell us very much, if only because neither continental philosophy nor post-modernism is much easier to characterize than it.

4.2 What it is

Nonetheless, even if none of these ideas picks out analytic philosophy precisely, each has some truth in it. There is at the very least a cluster if not of beliefs then of working methods which very many of those who regard themselves as analytic philosophers have held in common and which serve, when taken together, to illuminate something distinctive in their approach. We can insist first of all, I think, that the term 'analytic philosophy' is not wholly inappropriate: although there is no general agreement about what is analysed and why it is being analysed, the analytic method does nonetheless involve analysis.

The most prominent debate here concerns the holism of Quine, which has substituted the theory for the sentence as the appropriate unit on which surgery is to be performed. According to Quine, that is to say, it is midleading to attempt the analysis sentence-by-sentence. In explaning this idea Quine (1960, 3) famously adopted a metaphor of Neurath's according to which our theory is a ship which we must rebuild while staying afloat. (Quine himself called it a raft, perhaps to emphasize its fragility.)

Another leading idea has been the importance of rational argument in philosophy, not just as a tool but also as something which it is one of our primary tasks reflexively to critique and explain. I alluded earlier to the view, characteristic of a sort of naturalism, that rational norms just are scientific norms. A rather similar view, namely that rational argument just is logical argument, is nowadays even more widespread. I am not wholly convinced that this identification is correct, but what is at any rate clear is that it was only the developments in logic which began with Frege's invention of quantifier-variable notation in 1879 that made it even plausible. It is no accident, in other words, that analytic philosophy was born shortly thereafter.

Analytic philosophy may thus be seen as the inheritor of the 18th century debate between the rationalist and empiricist traditions. For part of the 20th century, indeed, analytic philosophers hoped that modern logic would close the gap between these two: rationalists, on this view, had appealed to reason as a source of knowledge distinct from sense experience only because they had thought of logic as essentially trivial; the power of modern logic re-awakened, for a time at least, the hope that some version of empiricism might give us if not all we want then at least all we need by way of knowledge.

Another common element in the analytic method has something to do with the ineluctability, when one argues from within a perspective, of the structural features of that perspective. But it is hard to formulate what this comes to in a way that all analytic philosophers would agree on. Perhaps the best formulation is Wittgenstein's: we must grant what he called the hardness of the logical 'must' (1953, §437)

The underlying point here goes well beyond logic. What is fundamental, not just in logic, is that there is a distinction between being true and being taken to be true. What exactly this distinction comes to has certainly not met with agreement among analytic philosophers. Nor is it even agreed whether it makes sense to suppose that our best theory of the world might be wrong: one sort of anti-realism consists precisely in denying this. Nonetheless, what analytic philosophers who present matters in terms of theory choice share is the view that there are criteria for the acceptance or rejection of a theory that are not wholly internal to the theory itself.

It is at the very least disputable, that is to say, whether it makes sense to suppose that we might all be wrong about everything; but it is not disputable that some of us may be wrong about something. Views which make errors impossible have surfaced from time to time, but only as proposals for dealing with specific problematic spheres of discourse (Wittgenstein in his middle period held such a view about arithmetic, for example) and certainly without much acclaim.

The final belief that many analytic philosophers hold in common is the one that Russell and Moore came to in their escape from Hegelianism, namely that the content of a judgement is not changed by the mere act of judging it. Not only is there a difference between being true and being taken to be true, but the latter does not change the former. (It is important, incidentally, to distinguish this from the superficially similar claim that coming to know something to be true does not change what is true. This last claim may well be false, as quantum mechanics tells us.) By means of their insistence on this point analytic philosophers aim to resist a sort of wholly general pessimism, prominent in the continental tradition, which concludes that we can never accurately represent anything about the world because the very act of representing ensures that we thereby miss our target. This no doubt contributes to the fact, noted earlier, that hostility to post-modernism has been especially strong among analytic philosophers.

4.3 Why it is

If we now have a sense of the dominant features that characterize the analytic movement, the further question then presses of why it has arisen. There are two facts to be explained here. On the one hand, the analytic tradition has achieved a dominance in English-speaking philosophy department that is, in its way, astonishing: in some departments authors in other traditions (Sartre, Derrida) are mentioned so rarely that undergraduates take their works to be a sort of pornography. On the other hand, the dominance is limited in both respects – only to philosophy departments, and largely (although this is now changing) to the English-speaking world.

As with other intellectual movements, some of the reasons for acceptance and rejection lie outside the discipline itself, in a jumble of historical, cultural and linguistic facts. An important factor contributing to the influence in America of logical positivism, for example, was the flight of (mainly Jewish) philosophers from occupied Europe in the 1930s. The lack of influence of some continental writers in Britain may have been partly a consequence of British linguistic incompetence. The approach to historical texts popular among analytic philosophers, of arguing with their authors on equal terms, and ignoring the awkward fact that Kant is not available to answer back, will from some cultural perspectives seem unduly disrespectful. The popularity of ordinary-language philosophy in Oxford after the Second World War was no doubt due partly to the fact that, unlike other approaches to philosophy then current, it could at least be practised competently by a Greats man without the least knowledge of modern science and mathematics.

One rather more internal factor in the acceptance of the analytic approach was undoubtedly its success: analytic philosophy made enormous progress in the fifty years after its birth, especially in the philosophy of mathematics, but also in the philosophies of language, mind and science. Once again, though, we cannot easily exclude external factors completely. The articles which Russell and Moore published in *Mind* during the 1900s are evidently more interesting and more illuminating than almost all of what surrounds them, but is that because of the power of the philosophical methods they had hit on or simply because they were able and inventive thinkers?

The analytic virtues of conceptual clarification and rational argument are no doubt applicable to problems that are not distinctively philosophical. (That, at any rate, is what we tell prospective philosophy students.) But the benefits the study of analytic philosophy is supposed to confer, of freeing the mind from prejudice and enabling it to see what is important in a problem, have their limits, as anyone who has attended a staff meeting in a philosophy department will attest. Moreover, it is noticeable that the successes of the method are much more prominent in some areas of philosophy than in others: analytic philosophy has told us much more in the last hundred years about the nature of mathematics and science than it has about art. For that reason it is perhaps no great surprise that literary critics have not on the whole been very interested in it.

The more general point lying in the background is this. Analytic thinking – thinking in accordance with the norms of analytic philosophy – may seem, to someone embedded in it, simply to be the same as clear thinking. The difficulty we have had in characterizing what analytic thinking involves might encourage the suspicion that this is not quite right. What I have tried to emphasize here is how the analytic method was developed at a particular time, in particular places, in response to particular problems. It may well be that some of what post-modernists say about the nature of the reader's response to a literary text is horribly confused, but it does not follow that anything analytic philosophy has to say about the matter, by being less confused, is thereby more illuminating.

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