Part IB Logic
Class 2: Metatheory of propositional calculus II
Prof Michael Potter
3pm Tuesday 4th February 2014
Mill Lane Room 10

Do not hand in your solutions in advance: write them out and bring them with you to the class. For explanations of the terminology used, consult the `Metatheory` lecture notes at `http://people.ds.cam.ac.uk/tecb2/teaching.shtml`.

Soundness

Read chapter 5 of `Metatheory`.

1. Show that the following are rule-sound:
   (a) $\vee$I
   (b) $\bot$E
   (c) $\rightarrow$I
   (d) $\rightarrow$E
2. Describe a way of enumerating the sentences of TFL, i.e. constructing an infinite list in which each sentence occurs once.

Completeness

Read chapter 6 of `Metatheory`.

3. Prove the following *without using the completeness theorem*.
   (a) If $\Gamma \vdash \mathcal{C}$ then $\Gamma \vdash \neg\neg\mathcal{C}$
   (b) If $\Gamma \vdash \mathcal{C}$ and $\Gamma \vdash \mathcal{D}$, then $\Gamma \vdash \mathcal{C} \wedge \mathcal{D}$
   (c) If $\Gamma \vdash \neg\mathcal{C}$ or $\Gamma \vdash \neg\mathcal{D}$, then $\Gamma \vdash \neg(\mathcal{C} \wedge \mathcal{D})$
   (d) If $\Gamma \vdash \neg\mathcal{C}$ and $\Gamma \vdash \neg\mathcal{D}$, then $\Gamma \vdash \neg(\mathcal{C} \vee \mathcal{D})$.
   (e) If $\Gamma \vdash \mathcal{A}$ and $\Gamma, \mathcal{A} \vdash \bot$, then $\Gamma \vdash \bot$
   (f) If $\Gamma \vdash \neg\mathcal{A}$ then $\Gamma, \mathcal{A} \vdash \bot$
   (g) If $\Gamma, \neg\mathcal{A} \vdash \bot$, then $\Gamma \vdash \mathcal{A}$
4. Prove Cases 5 and 6 of Lemma 6.10.
5. Show that if $\Gamma \vdash \mathcal{A}$ and $\Gamma, \mathcal{A} \vdash \mathcal{B}$ then $\Gamma \vdash \mathcal{B}$.
6. Let us say that Γ is *consistent* if $\Gamma \nvdash \bot$. Show that the following are equivalent:
   (a) $\mathcal{A}_1, \ldots, \mathcal{A}_n$ are consistent;
   (b) $\nvdash \neg(\mathcal{A}_1 \wedge \ldots \wedge \mathcal{A}_n)$;
   (c) $\nvdash (\mathcal{A}_1 \wedge \ldots \wedge \mathcal{A}_{n-1}) \rightarrow \neg\mathcal{A}_n$.