PHILOSOPHY TRIPOS, PART IA

Friday 2 June 2023 09.00–11:00

Paper 5

FORMAL METHODS

*Answer **ALL** questions in Section A.*
Each question in Section A is worth 9 Marks.

*Answer **TWO** questions from Section B.*
Each question in Section B is worth 20 marks.

Write the number of the question at the beginning of each answer

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION

Calculator – students are permitted to bring an approved calculator

Stationery Requirements
20 Page Answer Book
Rough Work Pad

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Part IA Paper 5 (Formal Methods)

Easter Term 2023

Section A

1. Which of the following claims are true, and which are false? Briefly explain your answers.
   (a) No valid argument has a contradictory conclusion.
   (b) If some sentences are jointly contrary, then there is a proof with those sentences as premises and an arbitrary sentence as conclusion.
   (c) For any TFL-sentences **A** and **B**, if **A** and **B** are not logically equivalent, then $\neg\mathbf{A}$ and **B** are logically equivalent.
2. Prove the following using the natural deduction system in *forallx:Cam*:
   (a) $P, Q \to (R \land \neg P), Q \lor S \vdash S$
   (b) $P \to (Q \leftrightarrow R) \vdash (P \land Q) \leftrightarrow (P \land R)$
   (c) $\exists x(Px \land \forall y Rxy) \vdash \exists z Rzz$
3. Consider the interpretation whose domain consists of the numbers 1, 2, 3, 4, 5 and 6, and for which the extensions of the predicates F and G are as follows:
   F is true of (and only of) 1, 2, and 3;
   G is true of (and only of) 4 and 5.
   Given this interpretation, state the truth values of the following sentences (you do not need to explain your answers):
   (a) $\forall x(Fx \lor Gx)$
   (b) $\exists x(Fx \land Gx)$
   (c) $\forall x(Fx \leftrightarrow \neg Gx)$
   (d) $\forall x \exists y(Fx \leftrightarrow \neg Gy)$
   (e) $\exists x \exists y(Fx \land Fy \land x \neq y)$
   (f) $\forall x \forall y((Gx \land Gy) \to x = y)$
4. Let A = {1, 2, 3, 4}, B = {1, 2} and C = {3, 5}. For each of the following statements say whether it is true or false; and if it is false say briefly why:
   (a) $A \subseteq B$
   (b) $A \cap C \subseteq A$
   (c) $\wp(B \cap C) \subseteq A \cap C$
   (d) $A \times B = B \times A$
   (e) $(A \times B) \cap (C \times B) = \varnothing$
5. Say of the following relations whether they are (i) reflexive (ii) symmetric (iii) transitive on the domain of all currently living people:
   (a) x and y are full siblings
   (b) x is the same age as y or the same height as y
   (c) x and y were once neighbours

Section B

6. (a) Use truth tables to demonstrate the following claims.
   i. $(B \to B) \to A \vDash (A \to B) \to (B \to A)$
   ii. $(A \to B) \to C \vDash A \to (B \to C)$
   (b) Explain why we can abbreviate $((A \wedge B) \wedge C)$ as $(A \wedge B \wedge C)$, but cannot abbreviate $((A \to B) \to C)$ as $(A \to B \to C)$.
   (c) Briefly explain the differences in meaning between $\to$, $\vDash$ and $\vdash$.
   (d) Prove that for any TFL-sentences $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A} \vDash \mathbf{B}$ if and only if $(\mathbf{A} \to \mathbf{B})$ is a tautology.
   (e) Is tautological entailment a transitive relation? Justify your answer (with appropriate use of metalinguistic variables).
   (f) Is the string "$(\mathbf{A} \to \mathbf{B})$" a sentence of TFL? If not, what is it?

7. (a) Consider the following symbolisation key:
   Domain: all people currently alive
   T: $\_\_\_\__1$ is taller than $\_\_\_\__2$
   P: $\_\_\_\__1$ has pink hair
   F: $\_\_\_\__1$ is friends with $\_\_\_\__2$
   a: Alice
   b: Bob
   Using this symbolisation key, give English-language versions of the following sentences.
   i. $\forall x(Fbx \to \neg Px)$
   ii. $\exists x\Big(\forall y\big(\forall z(y \neq z \to Tyz) \leftrightarrow x = y\big) \wedge Fax\Big)$
   iii. $\forall x(\exists y(Py \wedge Fxy) \to Tbx)$
   iv. $\forall x(Px \to \forall y(Fxy \to Py))$
   (b) Provide an interpretation—which can be independent of the above symbolisation—to show that the four sentences in part (a) are consistent.
   (c) Find three different ways to disambiguate the sentence "Only people who are taller than Bob and Alice's friends have pink hair." For each disambiguation, provide a symbolisation into FOL using the same symbolisation key as in part (a).

8. Let us call a relation R *Euclidean* if $\forall x \forall y \forall z\,((Rxy \wedge Rxz) \to Ryz)$. For each combination of properties (a)–(e) below, EITHER give an example of a relation combining those properties on a specified domain, OR explain why no such relation exists:
   (a) Transitive and Euclidean but not symmetric
   (b) Reflexive and symmetric but not Euclidean
   (c) Symmetric but neither Euclidean nor reflexive
   (d) Reflexive and Euclidean but not transitive
   (e) Symmetric, transitive and Euclidean but not reflexive

9. In two sacks of 100 coins each, 20 coins in each sack are biased in such a way that they always land heads; the other 80 coins in each sack are fair. You draw one coin from each sack at random: call one coin A and the other B. You then toss each coin three times. What is the probability that:
   (a) Coin A lands heads on its first toss?
   (b) Coin A lands the same on its first toss as coin B on its first toss?
   (c) Coin A lands the same on the first toss and *its* second toss?
   (d) Coin A lands heads on its third toss given that it landed heads on the first two tosses?
   (e) Coin A and coin B land the same as one another on their second toss given that they landed the same as one another on their first toss?

END OF PAPER