PHILOSOPHY TRIPOS, PART II

Tuesday 30 May 2023   13.30-16.30

Paper 7

MATHEMATICAL LOGIC

*Answer **three** questions only.*

Write the number of the question at the beginning of each answer.
If you are answering an either/or question, indicate the letter as well.

STATIONERY REQUIREMENTS
20 Page Answer Book x 1
Rough Work Pad

You may not start to read the questions
printed on the subsequent pages of this
question paper until instructed that you
may do so by the Invigilator

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1. Should Hilbert have cared more about soundness than about consistency?

2. EITHER: (a) If a thinker can be represented as a computable function at all, would the function necessarily be recursive and not primitive recursive?

    OR: (b) Could there be recursive functions that no person could ever know to be recursive? Justify your answer formally and describe any implications this has for whether minds are machines.

3. Outline a proof that every omega-consistent formal theory that extends basic arithmetic is incomplete. What is the significance of the use of 'omega-consistency' here?

4. What is achieved by a 'reduction' of mathematics to set theory?

5. 'The axiom of choice has obviously false consequences and must therefore be rejected.' Discuss.

6. Sketch a theory of transfinite cardinal numbers. Does it dissolve the paradoxes of the infinite?

7. Which is a better heuristic for solving Russell's paradox, the iterative conception or limitation of size?

8. Is there a viable semantics for second-order logic as the underlying logic of set theory?

9. EITHER: (a) Compare and contrast first-order Peano Arithmetic, first-order Complete Arithmetic, and second-order Peano Arithmetic. Is there any sense in which one is a better theory than the others?

    OR: (b) Compare the differences in expressive power of first-order logic (with and without identity) and second-order logic with respect to the cardinalities of models. Do these differences matter?

10. Say whether each of the following statements is true or false. Explain your answers, using examples as appropriate.

    a. The compactness of a logical system is a necessary condition for its being strongly axiomatizable.
    b. Full second-order logic is not compact.
    c. Every theory with an axiomatic presentation is effectively enumerable.
    d. Every axiomatizable, complete theory is decidable.
    e. The set of sentences of first-order logic with identity which are not logical truths is effectively enumerable.

END OF PAPER