PHILOSOPHY TRIPOS   Part IA

Tuesday 25 May 2004   9 to 12

Paper 3

LOGIC

Answer **four** *questions only; at least* **one** *from* **each section.**

Write the number of the question at the beginning of each answer.
Please answer **all parts** *of each numbered question chosen.*

> **You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator**

SECTION A

1   *(a)*   Explain what is meant by:
                (i)   a symmetric relation
                (ii)   a transitive relation
                (iii) a reflexive relation
                (iv) an equivalence relation.

    (b)   Let us say that a relation R is 'Euclidean' *iff*, for any x and y and z, if Rxy and Rxz then Ryz.
                (i)   Give an example of a Euclidean relation that is neither symmetric nor reflexive.
                (ii)   Show (by an informal argument or otherwise) that any transitive symmetric relation is Euclidean.
                (iii) Show (by an informal argument or otherwise) that any reflexive Euclidean relation is an equivalence relation.

2   Translate the following sentences into $QL^=$ (the language of the predicate calculus with identity), explaining the translation scheme you use.

    (a)   All athletes love Jacques.
    (b)   Some athletes who are philosophers are not logicians.
    (c)   Only an athlete loves a logician.
    (d)   Jacques, the well-known athlete, is not a logician.
    (e)   Whomever Kurt loves admires some logician.
    (f)   All philosophers love Kurt, but some of them also love Jacques.
    (g)   No logician loves everyone loved by Kurt.
    (h)   Only if Kurt is a logician is every student who is a philosopher a logician too.
    (i)   If Kurt loves athletes and Jacques is an athlete then Kurt does not love Jacques.
    (j)   Exactly three athletes love Kurt.

3   *(a)*   Explain *briefly* what a 'Dutch Book' Theorem is and what its significance is.

    (b)   You have £100 to spend on the horses.
            Three horses are running in the 2:15 at Taunton: A, B and C.
            A bookie offers you odds of 5-1 on A, 2-1 on B and 2-1 on C.
            Is there a way of distributing your bets so that you are bound to make money?
            What is the maximum return you can *guarantee*?

4   *(a)*   Define the following notions:
                (i)   *n*-place truth-functional connective.
                (ii)   expressive adequacy of a connective or set of connectives.

[TURN OVER for continuation of question 4]

(b)   The two-place truth-functional connectives ↑ and ↓ have the following truth tables:

p	q	p↑q		p	q	p↓q
T	T	F		T	T	F
T	F	T		T	F	F
F	T	T		F	T	F
F	F	T		F	F	T

Show that each of these connectives is expressively adequate.
(You may assume the expressive adequacy of the sets of connectives {∨,¬} and {∧,¬}.)

(c)   Show that ↑ and ↓ are the only expressively adequate two-place connectives.

5   Show that the following arguments are valid by translating them into QL⁼ (the language of the predicate calculus with identity) and using predicate trees:

(a)   Some logicians are athletes. All athletes are philosophers. So some logicians are philosophers.

(b)   Jacques is baffled by the predicate calculus. No logician is baffled by the predicate calculus. All mathematicians are logicians. So Jacques isn't a mathematician.

(c)   There is a sailor whom everyone loves. So everyone loves a sailor.

(d)   Some logicians admire anyone who has mastered the predicate calculus. Jones is a logician with no admirers. So Jones has not mastered the predicate calculus.

(e)   There are exactly two logicians. Akeel and Barbara are both logicians. Akeel and Barbara are not the same person. Jones is also a logician. So Jones is identical to either Akeel or Barbara.

(f)   The President of France loves the Queen of England. The President of France loves no one other than himself. The President of France and the Queen of England are not identical. Hence the moon is made of green cheese.

(g)   All logicians own cars. Any logician's car is a philosopher's car. No car is owned by more than one person. Hence all logicians are philosophers.

[TURN OVER]

SECTION B

6   What are the paradoxes of material implication? How would you solve them?

7   What is an a priori truth? What is a necessary truth? Must all a priori truths be necessary?

8   'The meaning of "The Queen of England" is the Queen of England.' Discuss.

9   Is analyticity a hopelessly confused notion?

10   What are the connections between logic and everyday reasoning?

END OF PAPER