PHILOSOPHY TRIPOS   Part IA

Tuesday 29 May 2007   09.00 to 12.00

Paper 3

LOGIC

Answer **three** *questions only; at least* **one** *from* **each section.**

Write the number of the question at the beginning of each answer. If you are answering the either/or question, indicate the letter as well.

STATIONERY REQUIREMENTS
20 Page Answer Book x 1
Rough Work Pad

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1   Attempt all parts of this question.

(a)   Using the following translation manual:

'a' for zero
'b' for one
'c' for two
'Px' for x is prime
'Ex' for x is even
'Ox' for x is odd
'Lxy' for x is less than y
'Gxy' for x is greater than y
'Sxyz' for x plus y equals z

and taking the domain of quantification to be the natural numbers (i.e. integers from zero up) translate the following sentences into QL with identity as best you can:

(i)   No even number is odd.
(ii)   Any number plus zero equals itself.
(iii)   If no number is less than zero then every number is either zero or greater than zero.
(iv)   For every number there is a greater one.
(v)   Every even number is the result of adding one to some odd number.
(vi)   If two numbers are distinct then their sum is not zero.
(vii)   There are at least two numbers.
(viii) The only even prime number is two.
(ix)   Two is the least number greater than zero and one.
(x)   Every even number greater than two is the sum of two prime numbers.

(b)   Use trees for QL with identity to show that the following are valid arguments.

(i)   All cricketers have good hand-eye coordination. No one clumsy has good hand-eye coordination. So no cricketer is clumsy.
(ii)   If Pingu is a penguin then some penguins are cute. Nothing cute is a carnivore. So if Pingu is a penguin then some penguins are not carnivores.
(iii)   Any true philosopher admires some logicians. Some students admire only existentialists and no existentialists are logicians. Hence not all students are true philosophers.
(iv)   There is a town to which all roads lead. So all roads lead to a town.
(v)   Angharad and Bethan, and they alone, love Caradoc. Someone who loves Caradoc kissed him. So either Angharad or Bethan kissed Caradoc.
(vi)   The donkey admired by Tim is not a carnivore. So no carnivore admired by Tim is a donkey.

[TURN OVER]

2   **Either**   *(a)*   What does it mean to say that the tree method for propositional
                             logic is 'sound' and 'complete'? Carefully prove that it is
                             complete, commenting on the strategy of the proof.

     Or   *(b)*   Explain the idea of a q-valuation. Explain how this idea
                             justifies the tree rules governing the universal quantifier.
                             Comment on the issue of empty domains.

3   Attempt all parts of this question.

     (a)   Set Theory

             (i)   Write down the axiom of extensionality and say what is meant by
                        the union $A \cup B$ and the intersection $A \cap B$ of sets A and B. What
                        does $A \subseteq B$ mean? What does $\wp(A)$ mean? What is $\varnothing$?
             (ii)   Show that if $A \subseteq B$ and $B \subseteq A$ then $A = B$.
             (iii)   Show that if $A \cap B = A$ then $A \subseteq B$.
             (iv)   Hence or otherwise show that: if $A \cap B = B$, and $B \cap C = C$, and
                        $A \cap C = A$, then $A = C$.
             (v)   Show that if $\wp(A) \subseteq A$ then $\wp(A) \in A$.

     (b)   Relations

             For each of the following relations say whether it is symmetric, reflexive or
             transitive. You may use the following information: that the author of
             Persuasion is identical to the author of *Mansfield Park* but distinct from the
             author of *Waverley*, and that there is no present King of France. The domain
             is the set of people.

             (i)   *x* is a brother of *y*.
             (ii)   *x* and *y* are brothers.
             (iii)   *x* was born in the same town as *y*.
             (iv)   *x* was born in the same town as *y* or *x* died in the same town as *y*.
             (v)   *x* wrote *Waverley* and *y* wrote *Persuasion*.
             (vi)   *x* wrote *Waverley* $\leftrightarrow$ *y* wrote *Persuasion*.
             (vii)   *x* wrote *Mansfield Park* $\leftrightarrow$ *y* wrote *Persuasion*.
             (viii)   If the present King of France loves *x* then the present King of France
                        loves *y*.
             (ix)   *x* is a brother of *y* $\supset$ *y* is a brother of *x*.
             (x)   *x* and *y* were married to the same person.

4   Attempt all parts of this question.

     (a)   Define conditional probability. Then use your definition where necessary to
             answer the following:

             Two cards are drawn at random and without replacement from a standard
             52-card pack without Jokers. Calculate the probability of the following
             events:

(i)   The first is a king.
(ii)   The first is a king and the second is a heart.
(iii)   The second is a spade given that the first is a spade.
(iv)   The first is a heart given that the second is a spade.
(v)   They are both aces given that one is an ace.
(vi)   They are both aces given that one is the ace of hearts.

(b)   In a small town 90% of taxis are yellow and 10% are green. One night a taxi is involved in a hit-and-run accident. An eyewitness says that the taxi is green. Tests later show that at night-time the eyewitness can identify yellow taxis as yellow 70% of the time and green taxis as green 80% of the time. What is the probability that the taxi involved in the accident was green?

SECTION B

5   What problems is Russell's Theory of Descriptions meant to solve? And is it the best way to solve them?

6   Give the strongest defence you can of the view that 'if ... then ...' in ordinary English has the same meaning as '⊃' in the propositional calculus. Is the defence adequate and if not why not?

7   Are all necessary truths knowable a priori?

8   'The words "sentence", "statement" and "proposition" are ambiguous in ways that are liable to lead to philosophical confusion.' Discuss.

END OF PAPER