

PHILOSOPHY TRIPOS Part IA

Tuesday 25 May 2010

09.00 to 12.00

Paper 3

LOGIC

*Answer **three** questions only; at least one from each section.*

Write the number of the question at the beginning of each answer.

STATIONERY REQUIREMENTS

20 Page Answer Book x 1

Rough Work Pad

**You may not start to read the questions
printed on the subsequent pages of this
question paper until instructed that you
may do so by the Invigilator**

SECTION A

1 Attempt all parts of this question.

(a) Carefully define the notions of

- (i) a truth-function
- (ii) a truth-functional connective
- (iii) an expressively adequate set of connectives
- (iv) tautological entailment

(b) Carefully explain the differences between what is symbolized by ' \supset ', and ' \vDash '.

(c) Show that $\{\vee, \neg\}$ is expressively adequate and $\{\vee, \supset\}$ isn't.

2 Attempt all parts of this question.

(a) Using the following translation manual:

- 'a' denotes Abelard
- 'e' denotes Eloise
- 'f' denotes Fulbert
- 'Sx' expresses: x is a student
- 'Cx' expresses: x is in a convent
- 'Px' expresses: x is a philosopher
- 'Lxy' expresses: x loves y

Taking the domain to be all people, translate the following into QL⁼.

- (i) Not every student in a convent is a philosopher.
- (ii) Eloise loves some philosopher only if all students are philosophers.
- (iii) Anyone who loves no philosophers does not love Abelard.
- (iv) Eloise loves at most one philosopher.
- (v) There are exactly two students whom Fulbert loves.
- (vi) If Eloise is in a convent, then Eloise is the only person Fulbert does not love; otherwise, the only person Fulbert does not love is Abelard.
- (vii) If exactly two philosophers are in a convent, then one of them is Eloise.

[continuation of question 2 on page 3]

- (b) Using the same translation manual, render the following arguments into $QL^=$ and use trees to show that they are valid.
- (i) No student is in a convent. The only philosophers there are are also students. So no philosopher is in a convent.
 - (ii) If Fulbert loves anyone, he loves exactly one person. Abelard is not a student. So if Fulbert loves Abelard, Fulbert loves no students.
 - (iii) Abelard loves Eloise. Eloise loves Abelard. Abelard is a philosopher. Fulbert loves no one who loves anyone who loves a philosopher. So Fulbert does not love Abelard.
 - (iv) Eloise loves Abelard and only Abelard. No one else loves anyone. So exactly one person is loved.

3 Attempt all parts of this question.

(a) Define what it means to say that:

- (i) A binary relation R is reflexive.
- (ii) A binary relation R is symmetric.
- (iii) A binary relation R is transitive.
- (iv) A binary relation R is an equivalence relation.

(b) Say that a binary relation R is *circular* iff $\forall x \forall y \forall z ((Rxy \wedge Ryz) \supset Rzx)$.

With this definition, prove the following claims, for any binary relation R .

- (i) Suppose R is circular and symmetric, and that everything bears R to something. Then R is reflexive.
- (ii) Suppose R is symmetric. Then R is circular iff R is transitive.
- (iii) R is reflexive and circular iff R is an equivalence relation.

(c) Let the domain of quantification be all people alive at the moment this logic examination started. For each of the following relations, say whether it is (1) reflexive, (2) symmetric, (3) transitive, and (4) circular. Where the answer is 'no', or a case could be made either way, explain your answer.

- (i) x and y share both parents.
- (ii) x and y are both female and share both parents.
- (iii) x is female and shares both parents with y .
- (iv) If x is female and shares both parents with y , then y is female and shares both parents with x .

[TURN OVER]

4 Attempt all parts of this question.

(a) Let A be the set of all women, B be the set of all Russians, and C be the set of all married Russians. Give the natural language translations of the following:

- (i) $C \subseteq (A \cap B)$
- (ii) $\text{Alexandra} \in (B \cup A)$
- (iii) $C \subset \emptyset (B)$
- (iv) $(A \cap B) \neq \emptyset$
- (v) $\text{Tatjana} \in (A / B)$
- (vi) $\{x: x \in A\}$

(b) What is the axiom of extensionality?

(c) Suppose that $X = \{\text{Ringo, John, Paul, George}\}$. And suppose that all and only the members of X are groovy. Show:

- (i) That there is no set of all the non-groovy things.
- (ii) That no member of $\wp(X)$ is groovy.
- (iii) That if Y is a subset of X then Y either has a groovy member or is the empty set.

(d) What is Bayes' Theorem?

(e) You are faced with two bags. Bag A contains 10 red balls, 9 of which have a black spot, and 2 unspotted white balls. Bag B contains 10 red balls, 1 of which has a black spot, and 50 unspotted white balls. You are passed one of the bags. You don't know which bag you have, though you know that there is a $\frac{1}{4}$ chance it is bag A, and a $\frac{3}{4}$ chance that it is bag B. What is:

- (i) The probability that you will pull a red ball out of the bag?
- (ii) The probability that you will pull a spotted ball out of the bag, given that you have bag B?
- (iii) The probability that the ball you will pull out will be spotted, given that it will be a red ball?
- (iv) The probability that you will first pull a white ball, followed by a red ball with a spot, given that you have bag A?
- (v) The probability that the ball you will pull out will be white, given that it will be a spotted white ball?

SECTION B

- 5 ' "The present King of France is bald" is neither true nor false.' Discuss.
- 6 Is the dichotomy between analytic and synthetic truths defensible?
- 7 With reference to Grice's notion of a conversational implicature, assess whether the material conditional of propositional logic provides an adequate translation of the English conditional.
- 8 What kind of truths are knowable a priori?

END OF PAPER