PHT0/3

PHILOSOPHY TRIPOS Part IA

Tuesday 29 May 2012 09.00 to 12.00

Paper 3

LOGIC

Answer **three** *questions only; at least* **one** *from* **each section.**

Write the number of the question at the beginning of each answer.

STATIONERY REQUIREMENTS
20 Page Answer Book x 1
Rough Work Pad

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Each question has equal weight. A perfect answer would receive a notional 100 points. For Part A [formal questions] the number in square brackets after each component of a question designates the number of points that a full and correct answer to that component would merit.

SECTION A

1 *(a)* Carefully define the following:

(i) validity; [5]
(ii) tautological validity; [5]
(iii) a tautology. [5]

(b) What is the difference between '$\therefore$', '$\models$', and '$\supset$'? [15]

(c) Can an inference be made invalid by adding extra premises? Give a reason for your answer. [15]

(d) The tree method for PL is sound and complete. Explain carefully what this means. [15]

(e) The connectives $\{\neg, \vee\}$ are expressively adequate. Explain what this means and prove that they are. [40]

2 *(a)* Carefully define the following terms in the context of the syntax and semantics of QL=:

(i) term; [5]
(ii) operator; [5]
(iii) scope; [10]
(iv) q-valuation; [10]
(v) extended q-valuation; [10]
(vi) q-validity. [10]

(b) Explain the semantics of the QL operators $\forall$ and $\exists$. Explain why any inference of the form $\neg\exists x F x, \therefore \forall x \neg F x$ is q-valid. [50]

3 *(a)* Let R be a relation. State the following conditions on R in the language of QL =:

(i) R is reflexive; [5]
(ii) R is symmetric; [5]
(iii) R is transitive. [5]

(b) Let us say that a relation R is *Euclidean* if $\forall x \forall y \forall z ((Rxy \wedge Rxz) \supset Ryz)$. Let us say that R' is the *converse* of R if $\forall x \forall y (Rxy \equiv R'yx)$.

Now translate the following arguments into QL = and show using trees that they are valid.

(i) if R is reflexive and symmetric and transitive then R is Euclidean; [15]
(ii) if R is symmetric and Euclidean and R' is the converse of R then R' is symmetric and Euclidean; [15]
(iii) if R is reflexive and R' is the converse of R then R' is reflexive. [15]

(c) Using the definitions in (b), give examples of relations that are:

(i) Euclidean and symmetric but not transitive; [10]
(ii) transitive and Euclidean but not symmetric; [10]
(iii) transitive and symmetric but not reflexive; [10]
(iv) not Euclidean but with a Euclidean converse. [10]

4 *(a)* You randomly draw two cards without replacement from a standard pack containing 52 cards: 13 of each suit (spades, hearts, diamonds, clubs) and no jokers. What is the probability that:

(i) the second is a spade? [5]
(ii) the first is a spade given that the second is *not* a spade? [10]
(iii) the second is a club given that the first is a heart? [10]
(iv) the second is a diamond given that the first is a diamond? [10]
(v) the second is a diamond given that the first is the king of diamonds? [15]

[TURN OVER for continuation of question 4]

(b) Billy is on trial for a crime. The DNA test shows that somebody with Billy's DNA was at the scene of the crime. 0.01% of the UK population shares Billy's DNA. So, the prosecution argues, there is a 99.99% probability that Billy was at the scene. Not so, says the defence: the UK population is 60 million, so there are 6,000 people with Billy's DNA, so the probability that Billy was at the scene is 1/6000. Who is right? [50]

SECTION B

5 Does 'The largest odd number is greater than 100' have a truth value?

6 Can there be synthetic necessary truths?

7 Explain *(a)* the type/token distinction and what is meant by 'an indexical expression' **and** *(b)* the relevance of these notions to logic.

8 What are the 'paradoxes of material implication'? How would you set about trying to resolve the concerns they might raise?

END OF PAPER