Each question has equal weight. A perfect answer would receive a notional 100 points. For Section A (formal questions), the number in square brackets after each component of a question designates the number of points that a full and correct answer to that component would merit.

SECTION A

1. This is a question about TFL. Attempt all parts of this question.
   - (a) Explain what these three sentences mean, and explain the differences between them:
     - $A \vDash C$
     - $A \vdash C$
     - $A \to C$ [15]
   - (b) State and prove the Disjunctive Normal Form Theorem. [50]
   - (c) Explain what it means to say, of some connectives, that they are jointly expressively adequate. Show that '$\wedge$' and '$\neg$' are jointly expressively adequate. You may rely upon your answer to part (b). [15]
   - (d) Are the connectives '$\wedge$', '$\vee$', '$\to$' and '$\leftrightarrow$' jointly expressively adequate? Explain your answer. [20]
2. Attempt all parts of this question. You **must** use the proof system from the course textbook, `forallx: Cambridge Version`.
   - (a) Show each of the following: [40]
     - (i) $\vdash (P \to Q) \vee (Q \to P)$
     - (ii) $\neg(P \leftrightarrow Q), \neg P \vdash Q$
   - (b) Show each of the following: [60]
     - (i) $\exists x(Fx \vee Gx) \vdash \exists xFx \vee \exists xGx$
     - (ii) $\forall x(Fx \to \forall yRxy), \forall x(Gx \to \forall zRxz), \forall x(\forall wRxw \to (Fx \wedge Gx)) \vdash \forall x(Fx \leftrightarrow Gx)$
     - (iii) $\forall x\exists yRxy, \exists x\forall y\, x = y \vdash \exists y\forall xRxy$
3. Attempt all parts of this question.
   - (a) Using the following symbolization key

     domain: all physical objects
     Mx: $\_\__x$ is a mug
     Rx: $\_\__x$ is red
     Tx: $\_\__x$ is a table
     Bxy: $\_\__x$ belongs to $\_\__y$
     a: Alice

symbolize each of the following sentences as best you can in FOL. If any sentences are ambiguous, or cannot be symbolized very well in FOL, explain why. [65]

(i) Every mug belonging to Alice is red.
(ii) The table is red.
(iii) Alice's mug is red.
(iv) Alice's mug does not exist.
(v) Two mugs are on the table.
(vi) If the mug belongs to anyone, it belongs to Alice.
(vii) None of the mugs on the table is Alice's.
(viii) Every mug is on exactly one table, and on every table there is exactly one mug.

(b) Show that each of the following claims is **wrong**. You may assume the conventions for representing interpretations described in the course textbook, `forallx: Cambridge Version`. [35]

(i) $Fa, \neg Ga, Fb, \neg Gb, \neg Fc, Gc \vDash \forall x(Fx \leftrightarrow \neg Gx)$
(ii) $\forall x(Fx \to \exists y(Gy \wedge Rxy \wedge \forall z((Gz \wedge Rxz) \to y = z))) \vDash$
$\forall x(Gx \to \exists y(Fy \wedge Ryx \wedge \forall z((Fz \wedge Rzx) \to y = z)))$
(iii) $\exists x \forall y \neg Ryx, \forall x \neg Rxx, \forall x \exists y Rxy \vDash \exists x \exists y(\neg x = y \wedge \exists z(Rxz \wedge Ryz))$

4. Attempt all parts of this question.

(a) Write down the axiom of extensionality. Then, using the standard notation, define the set-theoretic notions of: union, intersection, subset, proper subset and power set. [10]

(b) Give examples for each of the following: [10]

(i) Three non-empty sets, A, B, and C, such that none of $A \cap B$, $B \cap C$ and $A \cap C$ is empty, but such that $(A \cap B) \cap C$ is empty
(ii) Two different non-empty sets, A and B, such that $\wp(A) \cup \wp(B) = \wp(A \cup B)$

(c) Give examples for each of the following: [25]

(i) a set whose intersection with its power set is not empty
(ii) a set whose intersection with the power set of its power set is not empty
(iii) a non-empty set that is a subset of the power set of one of its members.

(d) Write down the axioms of probability. Explain conditional probability. [10]

(e) There are two equally probable hypotheses: either Bryce baked exactly 10 cupcakes today, or Bryce baked exactly 100 cupcakes today. In either case, Bryce piped unique numbers onto them: between 1 and 10, if there are exactly 10 cupcakes, or between 1 and 100, if there are exactly 100 cupcakes. Bryce hands you a cupcake, with the number 9 piped onto it. How probable is it, now, that Bryce baked exactly 100 cupcakes today? Explain your reasoning, highlighting any assumptions that you have made. [15]

(f) Attempt both parts of this question. [30]

(i) You are tossing a fair six-sided die. What is the probability that it lands 6 on each of the first three tosses? What is the probability that it lands 1, then 2, then 5?

(ii) Mr Corleone always chooses the same national lottery numbers. They came up in three successive lotteries, and now Mr Corleone is rich. But the lottery organizers are suing him for fixing the lottery. Mr Corleone's defence lawyer says: 'It is no more nor less likely, that these numbers should come up three times in a row, than that any other sequence of numbers should come up; so why should it be special grounds for suspicion?' Is there anything wrong with the lawyer's argument? Carefully explain your answer.

5. Attempt all parts of this question.

(a) Explain what it means to say that a relation is [5]

(i) reflexive
(ii) symmetric
(iii) transitive

(b) Say that a relation Rxy is Euclidean iff $\forall x \forall y \forall z((Rxy \land Rxz) \rightarrow Ryz)$. For each of the following relations on the domain of all people (living or dead), say whether the relation is reflexive, whether it is symmetric, whether it is transitive, and whether it is Euclidean. In each case that the relation fails to have one of these properties, briefly explain your answer. [50]

(i) x and y have the same surname
(ii) x and y have the same surname or the same first name
(iii) x loves y only if y loves x
(iv) x loves y iff y loves x
(v) x is Winston Churchill or y is Bertrand Russell
(vi) x is Winston Churchill iff y is Bertrand Russell

(c) Give examples of relations with the following properties. In each case, be careful to specify the domain. [45]

(i) Reflexive and transitive but not symmetric
(ii) Euclidean and transitive but not reflexive
(iii) Symmetric and transitive but not reflexive
(iv) Reflexive and symmetric but neither transitive nor Euclidean
(v) Neither reflexive, symmetric, transitive nor Euclidean

SECTION B

6. Does Russell's theory of definite descriptions provide a correct method of eliminating definition descriptions from all contexts in which they occur, from some contexts, or from none at all?
7. What, if anything, do the paradoxes of material implication tell us about the meaning of 'if…, then…' in natural language?
8. Are mathematical truths synthetic a priori?
9. Can meaning be explained in terms of intention?