SETS, RELATIONS AND PROBABILITY LECTURE 4

1. If R is an equivalence relation on a certain domain then there is a special *partition* on that domain. The notion of partition can informally be described like this: it is a way of dividing up the elements of a domain into sectors in such a way that every element belongs to exactly one sector. This means that the sectors are non-overlapping and include everything, or as we say: the division is exclusive (nothing belongs to more than one sector) and exhaustive (everything belongs to at least one sector). An example of a partition is the way the Allies divided up Berlin after WWII.

2. Slightly more formally, we can say that a partition is a set of such sectors, each sector itself being a set: that is: if X is a set then a partition on X is a set P of subsets of X such that:

   $\forall x (x \in X \to \exists p (p \in P \wedge x \in p \wedge \forall q ((x \in q \wedge q \in P) \to q = p)))$

3. These sectors are called R's *equivalence classes*. E.g. if R is the equivalence relation *x was born in the same country as y* then the equivalence classes are just these: the set of people born in the UK, the set of people born in France, … etc. If R is the equivalence relation on points on the surface of the Earth *x is the same height as y* then the equivalence classes are marked on a map as contour lines (that is why contour lines never meet). And Russell and Frege defined cardinal numbers as equivalence classes of *sets* on the equivalence relation *X and Y are equinumerous*. Question: how big are the equivalence classes of the identity relation?

4. I shall introduce a little more notation. If R is a relation then we say that the *converse* of R, sometimes written R', is that relation that holds between *x* and *y* if and only if R holds between *y* and *x*. I.e. R' is the converse of R iff $\forall x \forall y$ (R*xy* $\leftrightarrow$ R'*yx*). Thus e.g. *x is smaller than y* is the converse of *x is bigger than y*; and *x is above y* is the converse of *x is beneath y*. What do we call relations that are their own converses?

5. If R is a relation then the *ancestral* of R, sometimes written R*, is that relation that holds between *x* and *y* if and only if for some *z*, *w*, *v*, *u* ... we have R*xz* & R*zw* & … & R*vu* & R*uy*. For instance, if R is *x is a parent of y* then R* is *x is an ancestor of y*. What do we call relations that are their own ancestrals?

6. A straightforward application of the second point is to Locke's theory of identity. We know from Reid's objection that the memory-relation is not transitive. Plainly however identity is. Clearly then the relation Rxy = *x remembers being y (or vice versa)* can't be the same relation as *x and y are the same person*. But no such difficulty arises for R*.