SETS, RELATIONS AND PROBABILITY LECTURE 4

1. If R is an equivalence relation on a certain domain then there is a special partition on that domain. The notion of partition can informally be described like this: it is a way of dividing up the elements of a domain into sectors in such a way that every element belongs to exactly one sector. This means that the sectors are non-overlapping and include everything, or as we say: the division is exclusive (nothing belongs to more than one sector) and exhaustive (everything belongs to at least one sector). An example of a partition is the way the Allies divided up Berlin after WWII.

2. Slightly more formally, we can say that a partition is a set of such sectors, each sector itself being a set: that is: if X is a set then a partition on X is a set P of subsets of X such that:

\[ \forall x (x \in X \rightarrow \exists p (p \in P \land x \in p \land \forall q ((x \in q \land q \in P) \rightarrow q = p)) \]

3. These sectors are called R’s equivalence classes. E.g. if R is the equivalence relation x was born in the same country as y then the equivalence classes are just these: the set of people born in the UK, the set of people born in France, … etc. If R is the equivalence relation on points on the surface of the Earth x is the same height as y then the equivalence classes are marked on a map as contour lines (that is why contour lines never meet). And Russell and Frege defined cardinal numbers as equivalence classes of sets on the equivalence relation X and Y are equinumerous. Question: how big are the equivalence classes of the identity relation?

4. I shall introduce a little more notation. If R is a relation then we say that the converse of R, sometimes written R’, is that relation that holds between x and y if and only if R holds between y and x. I.e. R’ is the converse of R iff \( \forall x \forall y (Rxy \leftrightarrow Ryx) \). Thus e.g. x is smaller than y is the converse of x is bigger than y; and x is above y is the converse of x is beneath y. What do we call relations that are their own converses?

5. If R is a relation then the ancestral of R, sometimes written R*, is that relation that holds between x and y if and only if for some z, w, v, u … we have Rxz & Rzw & … & Rvu & Ruy. For instance, if R is x is a parent of y then R* is x is an ancestor of y. What do we call relations that are their own ancestrals?

6. A straightforward application of the second point is to Locke’s theory of identity. We know from Reid’s objection that the memory-relation is not transitive. Plainly however identity is. Clearly then the relation Rxy = x remembers being y (or vice versa) can’t be the same relation as x and y are the same person. But no such difficulty arises for R*.