SETS, RELATIONS AND PROBABILITY LECTURE 8

1. So far we have looked mainly at devices for calculating probabilities of events of interest on the basis of given probabilities for other events. We have considered the idea that probabilities might be interpreted as frequencies; but another interpretation of special interest is the following: the probability that you assign to an event E is a measure of your *degree of confidence* that E occurs.

2. On the face of it this attempts to put a precise quantitative interpretation on something that resists it. It is natural to think that your degree of confidence in something is a *feeling*: sometimes stronger, sometimes weaker, but hardly representable by a precise quantity between 0 and 1. But we can instead understand degrees of confidence as a degree of *belief*; and beliefs are propositional states and so perhaps more amenable to relatively precise behaviouristic analysis. The analysis that I have in mind is due to Frank Ramsey ('Truth and Probability in his *Philosophical Papers* ed. D. H. Mellor, and also in H. Smokler, ed., *Studies in Subjective Probability*).

3. In order to understand this analysis we need to know something about the odds given by bookies. If Ladbrokes offers, say, 1:1 on Labour winning the next election, that means that if you bet £50 on it and it happens you will get your £50 back plus another £50. If it doesn't happen then you lose your stake. Similarly, if they offer odds of 8:5 then of your £50 bet wins you will end up with £130 (£50 stake + £80 winnings). More generally, if they offer odds of A: B then if you win you will end up with £(A + B)/B for every pound that you stake. Now obviously some odds are better than others: for instance, odds of 10:1 are better (from your point of view) than odds of 5:1. Better odds are ones where (A+B)/B is higher.

4. An alternative system is known as American Odds. On this scheme the odds are given as a number: if the number is positive then it is how much you will win from a £100 stake; if it is negative then it represents how much you need to stake to win £100. Thus +800 represents better odds than +500; and -300 represents better odds than -400.

5. Now the degree of confidence that you have in an event E is determined by a behavioural disposition, namely the *worst* odds on E that you would be prepared to accept, at least when *small* stakes are involved. For instance, if the worst odds on E that you would be prepared to accept are 2:1 then that means that you have a degree of confidence of 1/3 in E. More generally, if the worst odds that you are prepared to accept is A:B then your confidence in E is B/(A+B). This makes intuitive sense: the more likely you think that something is to happen, the worse the odds at which you would bet on it. Thus if you are certain that something will happen then you will accept *any* bet on it at odds A:B (however small A is, as long as it is not zero); if you are certain that it will *not* happen you will accept *no* bet on it at odds A:B (however small B is, as long as it is not zero). In terms of American odds: if your lowest positive odds for E are +N then your confidence in

E is 100/(100 + N). The degree of confidence that you have in E is called your subjective probability or *credence* for E.

6. We started this course with an axiomatic presentation of a theory i.e. set theory. I wrote down certain axioms concerning what sets there are etc. and gave no more than the most intuitive justifications for them. In the case of probability the situation was similar. I wrote down certain axioms for the probability calculus and showed you how to derive, for a given event, a number that was called its *probability*. But for much of the course this was just a calculus, i.e. just the rules for playing a certain game with symbols. However, I didn't just pluck the axioms out of thin air. If the probability symbol Pr is interpreted in the subjectivist way, i.e. as a measure of your fair betting odds, then the axioms, and hence the theorems, of the probability theory have a kind of rational compulsion.

7. In order to see this we need to grasp the notion of a *Dutch Book*. We say that you are vulnerable to a Dutch Book if there is a set of betting odds on events that are arranged in such a way that you are both willing to take them and bound to lose money. Bookies as well as bettors can be vulnerable to Dutch books. Thus suppose for instance that Ladbroke's offers 3:1 on Red Rum, 3:1 on Shergar and 3:1 on Logic Boy, and suppose these are the only horses in the race. What you should do is put 1/3 of your capital on *each* of them: that way you are bound to win. Unfortunately bookies are aware of this fact and do not normally offer odds like this (or no *individual* bookie does).

8. The sense in which the axioms of probability are rationally compulsory depends on the point about Dutch Books. If we interpret probabilities subjectively then anyone who violates the axiom is vulnerable to a Dutch Book: that is to say, an intelligent punter could *always* make money out of them. For example, suppose that my confidence in E and $\neg$E respectively is Pr (E) = 0.75 and Pr ($\neg$E) = 0.75. Then clearly I have violated the theorem that Pr (X) + Pr ($\neg$X)=1.

9. Now why does this lead me into trouble? Well, it means that I will accept odds of 1:3 on E (say, rain tomorrow) and odds of 1:3 on $\neg$E. A cunning bookie will then simply offer me the chance to bet £3 on E and £3 on $\neg$E at those odds. I will take it. But then if E happens the bookie will lose £1 on the first bet and take £3 on the second; and if E does *not* happen the bookie will take £3 on the first bet and lose £1 on the second. Either way he has done me out of £2. The kind of economic irrationality that leaves me open to such exploitation is rather like what we saw with intransitive preferences. It is the avoidance of this kind of irrationality that is supposed to justify our accepting the laws of probability when these are interpreted as degrees of belief.