Each question has equal weight. A perfect answer would receive a notional 100 points. For Section A (formal questions), the number in square brackets after each component of a question designates the number of points that a full and correct answer to that component would merit.

SECTION A

1. Attempt both parts of this question.
   (a) Each of the following claims is either true or false:
      (i) $\vdash ((\neg B \to \neg A) \to A) \to A$ [15]
      (ii) $\neg E \to \neg\neg F, \neg G \to \neg E \vdash G \to \neg F$ [15]
      (iii) $(C \lor \neg A) \to (B \land D), C \to \neg B, \neg A \to \neg D \vdash A \land \neg C$ [15]
      (iv) $\neg(T \to S), (S \leftrightarrow (T \leftrightarrow U)) \vdash S \land \neg(T \leftrightarrow U)$ [15]
      (v) $C \leftrightarrow B, B \lor D, (C \lor D) \to A \vdash A$ [15]
      (vi) $P \leftrightarrow \neg Q, Q \leftrightarrow \neg R \vdash P \leftrightarrow R$ [15]

      For each true claim, show that it is true by providing a suitable formal proof (using the proof-system described in `forallx`). For each false claim, show that it is false by providing a suitable truth-table.
   (b) Explain the difference between the meanings of '$\vdash$' and '$\vDash$'. Explain, with reference to one of the false claims in part (a), why we are licensed in inferring its falsity from the truth table that you provided. [10]

2. Attempt all parts of this question.
   (a) Using the following symbolisation key

      domain: all people
      Nx: $\_\__x$ is a ninja
      Bxy: $\_\__x$ is behind $\_\__y$
      Sxy: $\_\__x$ can see $\_\__y$
      a: Akira

      symbolise each of the following sentences as best you can in FOL. Comment on your translations where appropriate, in particular highlighting any difficulties in symbolisation. [60]
      (i) Everyone behind Akira is a ninja.
      (ii) If Akira cannot see someone, that person doesn't exist.
      (iii) Only a ninja standing behind Akira is invisible to him.
      (iv) Akira is invisible to everyone except ninjas.
      (v) If one ninja stands behind another, the latter can see the former.
      (vi) The ninja Akira cannot see is behind him

(vii) No one can see the ninja behind Akira, not even that ninja herself

(viii) Behind every ninja Akira cannot see, there is another ninja Akira cannot see.

(ix) For every ninja Akira cannot see, there are two more ninjas Akira cannot see.

(x) Akira can see each of the three ninjas.

(b) Formalise these arguments, and then use the proof-system described in `forallx` to show that they are valid [40]

(i) If Akira cannot see someone, that person doesn't exist. The ninja is behind Akira. So Akira can see the ninja.

(ii) There is at least one ninja whom Akira cannot see. Behind every ninja Akira cannot see, there is another ninja Akira cannot see. So there are at least two ninjas whom Akira cannot see.

3. Attempt all parts of this question.

(a) Define the following set-theoretic notions: union, intersection, subset, power set, Cartesian product. [10]

(b) Each of the following statements is either true or false. In each case say which and explain briefly why: [45]

(i) For any sets A, B and C, if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

(ii) For any sets A and B, if $A \subseteq B$ then $A \subseteq \wp(B)$.

(iii) For any sets A and B, if $A \in \wp(B)$ then $\wp(A) \subseteq \wp(B)$.

(iv) For any sets A, B and C, $(A \cap B) \times (B \cap C) \subseteq A \times C$.

(v) For any set A, $\wp(A) \in \wp(\wp(A))$.

(c) Define the following notions from the logic of relations: reflexive, symmetric, transitive, equivalence relation. [10]

(d) Call a relation R negatively transitive if $\forall x \forall y \forall z((\neg Rxy \land \neg Ryz) \to \neg Rxz)$. Give examples of relations with each of the following properties: [35]

(i) equivalence, but not negatively transitive

(ii) reflexive and negatively transitive, but not symmetric

(iii) reflexive and symmetric, but not transitive

(iv) transitive and symmetric, but not reflexive

(v) transitive, but neither symmetric nor negatively transitive

4. Attempt all parts of this question.

(a) Define the terms: field, event space, conditional probability. [10]

(b) You are held captive in the Bayesian Republic of Zembla. The gaoler places two bullets in consecutive chambers in a six-chambered revolver, spins the wheel and takes a shot at you. You are in luck: the chamber was empty!

The gaoler then decides he will take a second shot. However, he decides to let you choose between the following options: *either* he will fire from the next chamber, *or* he will spin the chamber again and then fire. Which should you choose and why? [15]

(c) One octopus in every 100 is psychic. Psychic octopi have perfect knowledge of the results of future football tournaments. Non-psychic octopi can only guess randomly. Paul, a randomly chosen octopus, correctly predicts the winner of the next (football) World Cup, from the 32 teams that qualified. What is the probability that Paul is psychic? [25]

(d) I choose three different numbers, at random, from the (whole) numbers between 1 and 4 (inclusive). What is the probability that my choices are all in increasing (not necessarily consecutive) numerical order? [15]

(e) I choose three different numbers, at random, from the (whole) numbers between 1 and 10 (inclusive). What is the probability that they are in increasing (not necessarily consecutive) numerical order? [35]

SECTION B

5. Is 'the' a quantifier phrase?
6. Are there any contingent a priori truths?
7. '"If Rome is in the US, then it is in Europe" is not simply unassertible; it is also unbelievable. Hence the paradoxes of material implication cannot be explained away using the theory of implicatures.' Discuss.
8. Which should be considered primary in a philosophical account of meaning: sentence-meaning or speaker-meaning?