PHILOSOPHY TRIPOS   Part II

Thursday 28 May 2015   09.00 – 12.00

Paper 7

MATHEMATICAL LOGIC

Answer **three** *questions only.*

Write the number of the question at the beginning of each answer. If you are answering the either/or question, indicate the letter as well.

STATIONERY REQUIREMENTS
20 Page Answer book x 1
Rough Work Pad

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1. Say whether the following statements are true or false. Explain your answers, using examples where appropriate.

   (i) If a logic is strongly axiomatizable, it is compact.
   (ii) The property of having a finite domain is definable in first-order logic with identity.
   (iii) If a set of sentences of first-order logic without identity is satisfiable, then it has a denumerably infinite model.
   (iv) Every decidable, axiomatizable theory is complete.
   (v) Every axiomatizable, complete theory is decidable.

2. 'The natural framework for iterative set theory is second-order logic. But if the first-order variables range over all sets, then an instance of second-order comprehension says that there is a universal set, whereas iterative set theory says there is no such thing.' Solve this problem.

3. Explain why first-order Peano Arithmetic and first-order Complete Arithmetic have non-standard models, and why second-order Peano Arithmetic on its standard (full) semantics does not. Does this make second-order Peano Arithmetic a better theory?

4. What are the axioms of replacement and choice? Should we believe them?

5. Does set theory supply a foundation for mathematics? Does it need one?

6. Argue that every primitive recursive function is computable. Using a diagonal argument, show that there are computable functions that are not primitive recursive. Does a similar argument show that there are computable functions that are not recursive?

7. 'The Löwenheim-Skolem theorems have no philosophical significance whatever.' Discuss.

8. Does Gödel's second incompleteness theorem have any philosophical significance beyond that of his first incompleteness theorem?

9. EITHER: (a) Show that the set of numbers of the theorems of first-order Peano Arithmetic is not primitive recursive (assuming a primitive recursive numbering of the formulas of the theory's language).

   OR: (b) We can prove a completeness theorem for axiomatized first-order logic. So why can't we apply the same techniques to prove, contrary to Gödel's result, a completeness theorem for first-order logic extended with the first-order Peano axioms?

10. Sketch a proof of Tarski's theorem. What is its philosophical significance?

END OF PAPER