Mathematics and Reality

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Annotated Table of Contents

Chapter 1: Introduction (5,000 words)

The aim of this chapter is to set out the main structure of the book, and to show why the issue of the existence of mathematical objects provides a pressing problem for anyone seeking to understand natural science.

Part 1: An Ontological Proposal

Chapter 2: Naturalism and Ontology (10,000 words)

This chapter argues for taking a naturalist approach to ontology. That is to say, it argues that we should look to natural science to tell us what there is. This approach is defended against the charge that, allowing natural science to answer ontological questions conflicts with mathematical practice. The Quinean argument from this naturalistic starting point to mathematical realism is presented, and Hartry Field's response to this argument -- rejecting the indispensability of mathematics to natural science -- is briefly considered. It is concluded that we should for now accept that mathematics appears to be indispensable, and noted that the only further option open to an anti-realist would be to reject the Quinean account of ontological commitment.

2.1 The Naturalist Manifesto

The boat we're in: why we can only criticize our scientific theories from within. Science as our best effort to understand the world. The upshot for ontology: trust science to tell us what there is.

2.2 Scientific Naturalism and Mathematical Practice

The apparent conflict -- mathematics has its own methods for discovering mathematical truth and existence. (Parsons, Maddy.) This conflict is resolved by arguing that these concepts of mathematical 'truth' and 'existence' fall short of genuine existence and truth (since they are compatible with both Platonist and anti-Platonist accounts of mathematics). It is argued that the actual existence of mathematical objects is irrelevant to mathematical practice.

2.2 The Indispensability of Mathematics in Natural Science

Ontological Naturalism + Quine's criterion of ontological commitment + indispensable reference to mathematical objects in scientific theories = mathematical realism. Field's argument for dispensability; concerns about dispensability claim. What if mathematics is indispensable?

Chapter 3: Troubles with Indispensability (10,000 words)

In this chapter, the possibility of rejecting the indispensability argument from a naturalistic perspective is considered. Clearly, it is not enough for a naturalist to reject mathematical objects simply because they are suspicious of them -- this would be to abandon the commitment to trusting science. But recent work on the indispensability argument (particularly by Penelope Maddy) has suggested that it might be flawed for naturalistic reasons.

3.1 Does the Indispensability Argument Conflict with Mathematical Practice?

Various suggested points of conflict are considered and rejected. The suggestion of conflict is based on a misunderstanding of the goals and methods of pure mathematics. It is therefore argued that Maddy's (1997) move to adopt a parallel naturalism with respect to mathematics is both unnecessary and unwelcome.

3.2 Does the Indispensability Argument Conflict with Scientific Practice?

Examples of Maddy and others do appear to show that Quine's criterion for ontological commitment is not always in play in scientific practice, so there does appear to be conflict here.

3.3 Cracks in the Argument: Ontological Commitment Revisited

Quine's criterion for recognizing the ontological commitments of a discourse doesn't fit with scientific practices. Can we rethink our approach to ontology by looking for a more thoroughly naturalistic account of ontological commitment?

Chapter 4: Sceptical Worries (10,000 words)

Rethinking the issue of ontological commitment gives rise to the possibility of sceptical worries about the philosophical project of ontology. Quine's naturalist approach to ontology provided an answer to Carnap's anti-metaphysical ontological scepticism, but uncovering cracks in the Quinean picture leads to the possibility of a revived scepticism about the possibility of discovering 'what there is'. This chapter considers some recent sceptical worries about ontology, and points towards a way of resolving these worries by looking to scientific norms regarding explanation.

4.1 Will Any Approach to Ontology Beg the Question?

Jody Azzouni's argument to the effect that arguments about alternative approaches to ontology are bound to end in deadlock is considered and rejected.

4.2 Can We Separate Literal from Non-Literal Uses of Theories?

This section responds to Stephen Yablo's suggestion that our theories are so full of metaphors and idealizations that we will never be able to uncover the literal parts in those cases where ontological questions are at issue.

4.3 Is there any Coherent Ontology Lurking in our Scientific Theories?

Does recognizing the messiness of scientific theorizing, and in particular the lack of apparent unity between different parts of science, *show any naturalistic ontological project to be hopelessly over-optimistic? This section argues that it does not.*

Part 2: Carrying Out the Proposal

Chapter 5: Realism about Theories and Realism about Entities (10,000 words)

The shift in the second part of the book is to showing how the ontological project can be carried out. It is hoped that, by looking at the use of mathematics in our scientific theories, we will be able to show that it is reasonable for scientists to use mathematics and apparently refer to mathematical objects in their theories without believing in the existence of those objects.

A problem that immediately arises, however, is characterizing the correct attitude to scientific theories, if we no longer think that scientists should believe that their theories in their entirety are straightforwardly true. Mark Balaguer has suggested that we should believe a theory's physical content while remaining agnostic about its mathematical content, but it is difficult to unpack what is meant by the 'physical content' of a theory that is thoroughly mathematized. This chapter tackles this problem (of separating the mathematical from the physical content of a theory) by appealing to the method used by Bas van Fraassen to draw the distinction between the observable and unobservable content of a theory. While it is argued that van Fraassen's choice to draw a line between observable and unobservable objects is mistaken (on naturalistic grounds), his approach can still be used by those who would like to draw the line elsewhere. This opens the way for considering which objects we have reason to believe in, whether or not we fully believe our best theory of those objects. Thus the approach is similar to Ian Hacking's entity realism, according to which we can have reason to believe in the existence of objects without believing in the absolute truth of the theory that surrounds them.

5.1 The Separation Problem

Difficulties in the notion of 'physical content'. Problems with believing only parts of one's scientific theory.

5.2 Van Fraassen's Solution

The semantic approach to scientific theories.

5.3 Prospects for Entity Realism

We can make sense of the idea that it is the objects we are committed to, not the theories surrounding them. How can we now discover whether we are committed to mathematical objects?

Chapter 6: Explaining the Success of Mathematics (10,000 words)

An explanatory criterion for ontological commitment is presented and defended on naturalistic grounds. Looking at scientific practice shows that scientists see themselves as committed to the existence of objects when the hypothesis of the existence of those objects is required to explain the success of the theories in which they occur. In order to discover whether we are committed to the existence of mathematical objects, then, we need to consider how to explain the success of *mathematics in natural science. It is suggested that, whatever explanation we give of this success, it is unlikely that the hypothesis of realism about mathematical objects will do any work in this explanation.*

6.1 Success Arguments and Realism about Theories

Standard versions of such arguments say that the best explanation of the success of a theory is that it is true (and hence this covers the mathematical as well as the physical content of the theory). The assumption that the truth of the mathematical content explains its success is challenged.

6.2 Success Arguments and Realism about Entities

We should rather rethink our success arguments in terms of entity realism: in the case of electrons, we are led to realism because the success of our theory about electrons is best explained by the thought that there are objects - electrons - that behave roughly as our theory predicts they will behave. Hacking's experimental realism is considered as a possible variant of this kind of success argument.

6.3 Does Realism about Mathematical Objects Explain the Success of Mathematics?

How might hypothesizing the existence of mathematical objects explain the successful application of mathematics? Because mathematical objects are supposedly causally isolated, it is difficult to see how their existence is relevant. But we must hold back from drawing any conclusions until we can see what an explanation of the success of mathematics might look like.

Chapter 7: Kinds of Applications of Mathematics (10,000 words)

In order to look for an explanation of the success of mathematics, some different aspects of the application of mathematics in natural science are considered. It is shown that some applications can be dealt with fairly straightforwardly. However, the real test case is the application of mathematics in thoroughly mathematized scientific theories, such as modern physics.

7.1 Applying Mathematics to a Non-Mathematized Theory: Field's Account

The case of theories that can be expressed in non-mathematical terms. In this case, we need not believe in the truth of the mathematics applied in order to accept the conclusions of arguments about those theories which use mathematics, as Hartry Field has shown.

7.2 Worries about the Unreasonable Effectiveness of Mathematics in Natural Science: Wigner, Wilson, Steiner

We might still be puzzled about the fact that our mathematics turns out to be useful at all in these cases, and this is certainly a fact in need of explanation. However problems such as that of explaining why it is that the mathematics we choose to work on can turn out to have applications in areas that seem initially to be unrelated, though *important and interesting, do not appear to be relevant to the issue of commitment to mathematical objects.*

7.3 Theories that are Mathematical Through and Through: The Hard Case

The cases we really need to deal with, then, are those that are not covered by Field's explanation, since we cannot appeal to the conservativeness of mathematics. These are cases where the theories to which we apply mathematics cannot themselves be expressed except in mathematical terms. The suggestion (found, e.g., in papers by Balaguer and Yablo) that the mathematics is useful here because of its representational role is considered and endorsed, using the semantic view of theories to make sense of 'the physical content' that the mathematized theory represents. Our use of mathematics to represent the world as being in such-and-such a way does not commit us to the truth of the mathematics used in such representations.

Chapter 8: The Applicability of Mathematics in Quantum Physics (10,000 words)

In order to support the idea that the use of mathematics in thoroughly mathematized theories can be understood in terms of its representational role, this chapter looks at the development of quantum mechanics and considers the central role that the mathematics plays in this theory. Importantly, we have no underlying physical description of the 'reality' that the mathematics is being used to describe -- the theory is mathematical through and through. Nevertheless, it is argued that our use of mathematical representations here at the ground level is not ontologically committing.

8.1 Historical Development

Von Neumann, Dirac, Weyl…
How did the use of abstract mathematics in quantum mechanics get off the ground?

8.2 Alternative Mathematical Representations

We can do quantum mechanics using groups, or using Hilbert spaces. Otavio Bueno has asked, if the two uses of mathematics are interchangeable, what should a traditional indispensability theories take themselves to be committed to?

8.3 A Case Study

The C-algebra approach to quantum field theory.*

Chapter 9: Conclusion (5,000 words)

Some consequences of the argument are drawn out.

9.1 Agnosticism or Atheism?

It has been argued that the best explanations of the uses of mathematics in natural science will not require the hypothesis of realism about mathematical entities. Does this mean that we should *reject mathematical platonism, or simply remain agnostic about the existence of mathematical objects? To the extent that we have accepted the naturalistic project of ontology, we should reject mathematical platonism: if we take it that natural science tells us what we have reason to believe, then we should conclude, as Hartry Field has put it, that we have no more reason to believe in mathematical objects than we have to believe in 'little green people living inside electrons and that are in principle undiscoverable by human beings' -- and here, agnosticism rather than atheism seems undue epistemological caution.*

9.2 Levels of Commitment to Physical Objects

Are we committed to the existence of all the physical objects referred to in our theories? Not at all -- in many cases the best explanation of the success of a theory that mentions physical objects will not require that those objects exist (e.g. in the case of explicit idealizations). The line drawn is not a simple mathematical/physical one -- instead ontology should be done on a case by case basis. However, the general form of explanations of the use of mathematics in natural science suggests that in this case, no such explanation will require the hypothesis of realism there.

9.3 What of Mathematical 'Truth'?

The upshot is a 'fictionalist' approach to mathematics. But can we speak with the vulgar and still talk of mathematical results as being 'true' and even 'certain'? Certainly -- we might even allow that there is a species of mathematical truth, different from ordinary truth -- but that mathematics is true in this weak sense has no bearing on the issues of ontology or applicability.