PHT2/7

PHILOSOPHY TRIPOS Part II

Monday 31 May 2004 9 to 12

Paper 7

MATHEMATICAL LOGIC

Answer **three** *questions only.*

Write the number of the question at the beginning of each answer. If you are answering the either/or question, indicate the letter as well.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 Sketch a proof of the completeness of first-order logic with identity.

2 Compare and contrast the expressive power of first-order logic without identity, first-order logic with identity, and second-order logic. Are the differences philosophically significant?

3 Show how to construct an arithmetic sentence G which says of itself that it is not provable in Peano arithmetic. Show that adding $\neg G$ to Peano arithmetic results in an ω-inconsistent theory. Explain how this entails the existence of non-standard models of Peano arithmetic.

4 **Either** *(a)* Does Gödel's second incompleteness theorem refute Hilbert's programme?

Or *(b)* Do Gödel's incompleteness theorems imply that the mind cannot be represented by a formal system?

5 State and prove Dedekind's categoricity theorem for second-order Peano arithmetic. Assuming Gödel's first incompleteness theorem, show that any sound deductive system for second-order logic must be incomplete.

6 What is the Skolem paradox? Do categoricity theorems help explain how we can refer to the standard model of arithmetic?

7 'Second-order logic is set theory in sheep's clothing.' Discuss.

8 Does the iterative conception of set lead to a satisfactory resolution of all the set-theoretic paradoxes?

9 Do the mathematical consequences of the axiom of choice give us any reason to think it is true?

10 What can we learn from Gentzen's consistency proof for Peano arithmetic?

END OF PAPER