PHILOSOPHY TRIPOS   Part II

Wednesday 30 May 2007   09.00 to 12.00

Paper 7

MATHEMATICAL LOGIC

Answer **three** *questions only.*

Write the number of the question at the beginning of each answer. If you are answering an either/or question, indicate the letter as well.

STATIONERY REQUIREMENTS
20 Page Answer Book x 1
Rough Work Pad

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1   'The iterative conception of set is the only one that explains the
       contradictions.' Discuss.

2   Is the axiom of choice true?

3   Is countability a relative notion?

4   What is meant by saying that the proof of Cantor's theorem is impredicative?
       Does this impredicativity give us any reason to doubt the truth of the
       theorem?

5   **Either**   *(a)*   Compare the expressive power of
                            (i)   first-order logic without identity,
                            (ii) first-order logic with identity, and
                            (iii) second-order logic.

       Or   *(b)*   Is second-order logic set theory in disguise?

6   What is a maximal consistent, omega-complete set of sentences? Explain how
       such sets can be used in proving the completeness of a deductive system for
       first-order logic without identity. Show how the (downward)
       Löwenheim-Skolem theorem follows as a corollary.

7   Compare and contrast the relations between
                            (i)   first-order Peano Arithmetic,
                            (ii) the set of first-order arithmetical truths,
                            (iii) second-order Peano Arithmetic, and
                            (iv) the set of second-order arithmetical truths.

8   Carefully state one version of Gödel's first incompleteness theorem,
       explaining the technical terms that you use. Explain a strategy for proving it.

9   **Either**   *(a)*   What, if anything, does Gödel's first incompleteness theorem
                            show about our understanding of quantification over
                            numbers?

       Or   *(b)*   After Gödel, can anything survive of Hilbert's programme?

10   **Either**   *(a)*   Outline two different formal accounts of what it is for a
                            function to be computable. Discuss the strategy for showing
                            the two accounts are extensionally equivalent.

       Or   *(b)*   How compelling are the reasons for accepting the
                            Church-Turing Thesis?

END OF PAPER