PHT2/7

PHILOSOPHY TRIPOS Part II

Friday 21 May 2010 09.00 to 12.00

Paper 7

MATHEMATICAL LOGIC

Answer **three** *questions only.*

Write the number of the question at the beginning of each answer. If you are answering an either/or question, indicate the letter as well.

STATIONERY REQUIREMENTS
20 Page Answer book x 1
Rough Work Pad

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 What can be said for and against second-order logic as a basis for mathematics?

2 Which first-order theories can be categorical? Which cannot be? Why does categoricity matter?

3 'Every maximal consistent, ω-complete set of sentences has a model.' Explain what this means. Outline a proof of it for a system of first-order logic.

4 What is a primitive recursive function? Outline a proof that first-order Peano Arithmetic can express all primitive recursive functions.

5 **Either** *(a)* Explain carefully what Gödel's first incompleteness theorem says. Outline your favourite proof.

Or *(b)* Show how to construct an arithmetical sentence G which says of itself that it is not provable in first-order Peano arithmetic. Show that adding $\neg G$ to first-order Peano arithmetic results in an ω-inconsistent theory. Explain how this entails the existence of non-standard models of the original theory.

6 Does Gödel's second incompleteness theorem refute Hilbert's programme?

7 'Church's thesis is in principle refutable but not provable.' Discuss.

8 **Either** *(a)* 'The iterative conception is the only natural and (apparently) consistent conception of set we have.' Discuss.

Or *(b)* 'Although they are not derived from the iterative conception, the reason for adopting the axioms of replacement is quite simple: they have many desirable consequences and (apparently) no undesirable ones.' Discuss.

9 Outline an account of how arithmetic and analysis can be embedded in set theory.

10 Outline the main ideas of the arithmetic of infinite cardinal numbers. State and prove Cantor's theorem. Explain in what sense the proof of the theorem is impredicative.

END OF PAPER