

PHILOSOPHY TRIPOS Part II

Friday 27 May 2011

09.00 to 12.00

Paper 7

MATHEMATICAL LOGIC

*Answer **three** questions only.*

Write the number of the question at the beginning of each answer. If you are answering an either/or question, indicate the letter as well.

STATIONERY REQUIREMENTS

20 Page Answer book x 1

Rough Work Pad

**You may not start to read the questions
printed on the subsequent pages of this
question paper until instructed that you
may do so by the Invigilator**

- 1 **Either** (a) Is second-order Peano Arithmetic any improvement on its first-order counterpart?
Or (b) Explain some of the technical and philosophical problems in giving a semantics for second-order quantification.
- 2 How paradoxical is Skolem's Paradox?
- 3 Describe a system of first-order logic. Outline a proof that it is strongly complete.
- 4 Should we believe the Axiom of Foundation?
- 5 What does the possibility of embedding arithmetic and analysis in set theory show?
- 6 Outline an account of ordinal numbers and their arithmetic. Discuss examples of arithmetical laws which hold for the natural numbers but not for the ordinal numbers in general.
- 7 **Either** (a) Distinguish the semantic and syntactic versions of Gödel's First Incompleteness Theorem. Outline a proof of the semantic version for first-order Peano Arithmetic.
Or (b) What is Gödel's Second Incompleteness Theorem? How is it proved?
- 8 What is the halting problem? Sketch a proof that it is unsolvable. What is the significance of this result?
- 9 **Either** (a) Outline two different formal accounts of what it is for a function to be computable. Discuss a strategy for showing that the two accounts are extensionally equivalent.
Or (b) Are there any good reasons for doubting the Church-Turing Thesis?
- 10 What is Hilbert's Programme? How defensible is it?

END OF PAPER