

PHILOSOPHY TRIPOS Part II

Thursday 31 May 2012

09.00 to 12.00

Paper 7

MATHEMATICAL LOGIC

*Answer **three** questions only.*

Write the number of the question at the beginning of each answer. If you are answering the either/or question, indicate the letter as well.

STATIONERY REQUIREMENTS

20 Page Answer book x 1

Rough Work Pad

**You may not start to read the questions
printed on the subsequent pages of this
question paper until instructed that you
may do so by the Invigilator**

- 1 Attempt all parts of this question:
 - (a) What is compactness? Comment on its logical and philosophical significance.
 - (b) Which of these is compact: first-order logic without identity, first-order logic with identity, second-order logic? Give counterexamples to compactness where appropriate.
- 2 Do the Löwenheim-Skolem theorems have any philosophical significance?
- 3 Compare and contrast first-order and second-order Peano Arithmetic. Is one to be preferred?
- 4 Attempt all parts of this question:
 - (a) State and prove Cantor's theorem. Explain in what sense the proof of the theorem is impredicative.
 - (b) Is there a good answer to the question 'how many cardinal numbers are there?'
- 5 **Either** (a) What is the Axiom of Choice? Is it true?
Or (b) 'The iterative conception of set is the only one that explains the set-theoretic paradoxes.' Discuss.
- 6 Is the rest of mathematics set theory in disguise?
- 7 Let T be a consistent, axiomatized formal theory. Explaining all technical terms, show all of the following:
 - (a) if T is negation-complete, it is decidable;
 - (b) if T is sufficiently strong, it is undecidable;
 - (c) if T is sufficiently strong, it is not negation-complete;
 - (d) if T is sufficiently strong, finitely axiomatized, and based on first-order logic, then first-order logic is undecidable.
- 8 Attempt all parts of this question:
 - (a) Explain the following ideas: effectively computable total function; primitive recursive function; recursive function.

[continuation of question 8 on page 3]

(b) Using a diagonal argument, show that there is an effectively computable total function which is not primitive recursive.

(c) Explain why Church's thesis entails that there is no effective procedure for determining whether a string is a definition of a recursive function.

9 Attempt all parts of this question:

(a) Construct a Gödel sentence G for first-order Peano Arithmetic. Using a syntactic argument, outline a proof that the theory does not decide G .

(b) Is G true? Explain your answer.

10 Do Gödel's incompleteness theorems tell us anything about the human mind?

END OF PAPER