

PHILOSOPHY TRIPOS Part II

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Friday 24 May 2019

09.00 – 12.00

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Paper 7

MATHEMATICAL LOGIC

Answer **three** questions only

*Write the number of the question at the beginning of each answer. If you are answering an either/or question, indicate the letter as well.*

**STATIONERY REQUIREMENTS**

*20 page answer book × 1*

*Rough work pad*

**You may not start to read the questions  
printed on the subsequent pages of  
this question paper until instructed that  
you may do so by the Invigilator**

1. Expound a semantics for a system of second-order logic. Does the semantics show that second-order logic is set theory in disguise?
2. Sketch a Henkin-style proof of the completeness of first-order logic with identity, and show how both the compactness of the logic and a Löwenheim–Skolem theorem follow as corollaries.
3. Is there any sense in which set-theoretic notions are relative? Why does it matter?
4. Give axioms for first-order and second-order Peano arithmetic. Explain whether the resulting theories are (i) decidable, (ii) complete, (iii) categorical.
5. EITHER: (a) Outline a theory of transfinite ordinal numbers.  
  
OR: (b) Can the question ‘How many ordinal numbers are there?’ be answered?
6. EITHER: (a) Can the Axiom Scheme of Replacement be justified?  
  
OR: (b) Can the Axiom of Choice be justified?
7. Argue that every primitive recursive function is computable. Show that there are computable functions that are not primitive recursive.
8. ‘Church’s Thesis cannot be proven but it might be refuted.’ Discuss.
9. EITHER: (a) Show that any axiomatized theory, the language of which can express all the recursive properties of the natural numbers, is either unsound or incomplete.  
  
OR: (b) ‘The Gödel sentence is true and unprovable only if it means “I am not provable”, and it has this meaning only if numbers are taken to represent sequences of symbols. Since numbers should not be taken in that way, Gödel’s first incompleteness theorem has not been proven.’ Do you agree?
10. Do Gödel’s incompleteness theorems show us anything about the mind?

**END OF PAPER**