PHT0/3

PHILOSOPHY TRIPOS Part IA

Tuesday 24 May 2015 09.00 – 12.00

Paper 3

LOGIC

Answer **three** *questions only; at least* **one** *from* **each** of **sections A and B.**

Write the number of the question at the beginning of each answer.

Each question has equal weight. A perfect answer would receive a notional 100 marks. For Section A (formal questions) the number in square brackets after each component of a question designates the number of marks that a full and correct answer to that component would merit.

STATIONERY REQUIREMENTS
20 Page Answer Book x 1
Rough Work Pad

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Section A

1. (a) Show each of the following using the proof system for TFL described in *forallx*.

(i) $\neg(P \leftrightarrow Q) \vdash \neg(Q \leftrightarrow P)$
(ii) $(P \wedge \neg Q) \vee R \vdash Q \rightarrow (P \vee \neg\neg R)$
(iii) $\neg\big((P \rightarrow \neg Q) \vee \neg R\big) \vdash Q$
(iv) $\neg\big(P \rightarrow (Q \vee R)\big) \vdash P \wedge \neg R$
(v) $\vdash P \leftrightarrow ((P \wedge \neg Q) \vee (P \wedge Q))$ [50]

(b) Show each of the following using the proof system for FOL described in *forallx*.

(i) $\forall x \forall y\ x = y \vdash \neg\exists x \exists y \neg x = y$
(ii) $\forall x \neg Rxx, Rab \vdash \exists x \exists y \neg x = y$
(iii) $\forall x \exists y (Ryx \vee Qyx) \vdash \forall y (\exists x Ryx \vee \exists x Qyx)$
(iv) $\forall x \exists y Rxy,\ \exists x \exists y Rxy \vdash \exists x \exists y \exists z (Rxy \wedge Ryz)$
(v) $\exists x\big(Fx \wedge \forall y(Fy \rightarrow y = x)\big), \forall x(Fx \leftrightarrow Gx) \vdash \exists x(Gx \wedge \forall y(Gy \rightarrow y = x))$

[50]

2. Answer all parts of this question.

(a) Using the following symbolisation key

Domain: the Muppets
Fx: ___$_x$ is a frog
Jx: ___$_x$ tells jokes
Sx: ___$_x$ is a singer
Fxy: ___$_x$ is funnier than ___$_y$
Lxy: ___$_x$ loves ___$_y$
g: Gonzo
k: Kermit
m: Miss Piggy

symbolise the following English sentences as best you can in FOL, commenting on any difficulties or limitations you encounter:

(i) Miss Piggy loves Kermit unless Gonzo is funnier than Kermit.
(ii) The frog whom Miss Piggy loves also tells jokes.
(iii) Precisely two singers are loved by Kermit.
(iv) There are at most three singers, who tell jokes, and none of them are frogs.
(v) A frog's jokes are always funnier than a singer's.
(vi) Both singers and frogs tell jokes.

(vii) All frogs love Miss Piggy, and some love Gonzo too.
(viii) Given that Kermit tells jokes, some frog must tell jokes.
(ix) Some frog and some singer are exactly as funny as Gonzo.
(x) Three singing frogs are funnier than Miss Piggy. [50]

(b) Show that each of the following claims is true.

(i) $\forall y(\exists x Ryx \wedge \exists x Qyx) \nvDash \forall y \exists x(Ryx \wedge Qyx)$
(ii) $\exists x(Px \rightarrow Qx) \nvDash \exists x Px \rightarrow \exists x Qx$
(iii) $\forall x\big(Px \rightarrow \exists y(Rxy \wedge Ryx)\big), \neg\exists x Rxx \nvDash \forall x \neg Px$
(iv) $\exists x \forall y(Fy \leftrightarrow x = y), \exists x \forall y(Gy \leftrightarrow x = y)$
$\nvDash \exists x \exists y \Big(\neg x = y \wedge \forall z\big((Fz \vee Gz) \leftrightarrow (x = z \vee y = z)\big)\Big)$
[40]

(c) Explain the differences between what is symbolized by '$\vdash$' and '$\vDash$'. [10]

3. (a) Explain the axiom of extensionality. Let R be the set of numbers {1, 2, 3, 4}, let S be the set {3, 4, 5, 6} and let T be {5, 6, 7, 8}. Write down the elements of the following sets:

(i) $R \cup T$
(ii) $R \cap (S \cup T)$
(iii) $\wp(R \cap S)$
(iv) $\wp(R) \cap \wp(S)$
(v) The set Y of all elements of S that are greater than some element of T – S. [20]

(b) Define reflexivity, symmetry and transitivity. For each of the following relations, say which of these properties it possesses (on the domain of presently living people):

(i) *x* is at least one inch taller than *y*
(ii) *x* is at most one inch taller than *y*
(iii) *x* is taller than y iff y is taller than *x*
(iv) *x* is exactly 80 years older than *y*
(v) *x* is a brother of *y*
(vi) *x* and *y* are brothers
(vii) *x* and *y* have the same first name or the same last name
(viii) *x* owns a helicopter only if *y* does
(ix) *x* is David Hasselhoff and *y* is not
(x) Most people prefer *x* to *y* [80]

4. (a) Write down the probability axioms and the definition of conditional probability. [10]

(b) I draw three cards at random without replacement from a standard pack of cards. What is the probability:

(i) That the first two are aces? [5]
(ii) That the first two are aces given that the first is an ace? [10]
(iii) That the first two are aces given that the third is the King of Hearts? [15]

(c) I have two bags. One of them contains 50 red balls and 50 black balls. The other contains 60 red balls and 40 black balls. I cannot tell which is which. I choose a bag at random and draw a ball from it (without putting it back). It is red. Then I draw another ball from the same bag. What is the probability that this second ball is red? [30]

(d) Four balls are placed in a bowl. One is green, one is black and the other two are yellow. The bowl is shaken and someone draws two balls from the bowl. He looks at the two balls and announces that at least one of them is yellow. What is the probability that the other ball he has drawn out is also yellow? [30]

Section B

5. How does Russell's theory of definite descriptions help solve the problem of negative existentials?

6. Is it essential to conditionals that they support *modus ponens*?

7. EITHER (a) Is '2+2=4' a synthetic *a priori* truth?

   OR (b) Is 'water=H_2O' a necessary *a posteriori* truth?

8. Is sentence-meaning reducible to speaker-meaning? Is speaker-meaning reducible to sentence-meaning?

END OF PAPER