

PHILOSOPHY TRIPOS Part IA

Tuesday 30 May 2017

09.00 – 12.00

Paper 3

LOGIC

*Answer **three** questions only; at least one from each of sections A and B.*

Write the number of the question at the beginning of each answer. If you are answering the either/or question, indicate the letter as well.

Each question has equal weight. A perfect answer would receive a notional 100 marks. For Section A (formal questions) the number in square brackets after each component of a question designates the number of marks that a full and correct answer to that component would merit.

STATIONERY REQUIREMENTS

20 Page Answer Book x 1

Rough Work Pad

<p>You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator</p>

Section A

1. (a) Show each of the following using the proof system for TFL described in *forallx*.

- (i) $P \leftrightarrow \neg(P \vee Q) \vdash P \rightarrow R$
- (ii) $\neg(P \rightarrow Q) \vdash P$
- (iii) $\vdash (P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$
- (iv) $\neg(P \rightarrow (Q \vee R)) \vdash P \wedge \neg Q \wedge \neg R$
- (v) $\neg((P \wedge Q) \leftrightarrow R) \vdash (\neg P \vee \neg Q) \leftrightarrow R$ [50]

(b) Show each of the following using the proof system for FOL described in *forallx*.

- (i) $\exists x \neg Pxa \vdash \exists z \neg \forall y Pyz$
- (ii) $\exists x \exists y \forall z \forall w Pxyzw \vdash \forall z \exists x \forall w \exists y Pxyzw$
- (iii) $\forall x \exists y (x = a \leftrightarrow Raay), \forall x (Px \vee \neg \exists y Rxy) \vdash Pa$
- (iv) $\forall x \exists y Rxy, \forall x \forall y (Rxy \rightarrow Ryx), \forall x \forall y \forall z ((Rxy \wedge Ryz) \rightarrow Rxz) \vdash \forall x Rxx$
- (v) $\exists x \forall y (Fy \leftrightarrow x = y), \exists x \forall y (Gy \leftrightarrow x = y), \neg \exists x (Fx \wedge Gx) \vdash \exists x \exists y (\neg x = y \wedge \forall z ((Fz \vee Gz) \leftrightarrow (x = z \vee y = z)))$ [50]

2. Answer all parts of this question.

(a) Using the following symbolisation key

Domain: jazz musicians

Px : $___x$ plays the piano

Sx : $___x$ plays the saxophone

Mx : $___x$ is a musician

Dxy : $___x$ has played in a duo with $___y$

$Txyz$: $___x$ has played in a trio with $___y$ and $___z$

b : Brad

j : Joshua

p : Pat

symbolise the following English sentences as best you can in FOL, commenting on any difficulties or limitations you encounter:

- (i) Every musician has played in a duo with someone.
- (ii) Brad has played in a duo with Joshua but sometimes duets with others.
- (iii) The pianist who has played in a duo with Pat is not Brad.
- (iv) There are two musicians who have played in a trio with Brad and Pat other than Joshua.
- (v) The saxophonist who has played in a duo with Pat cannot be the pianist who has played with Joshua.
- (vi) No one has played in a trio with Brad and Joshua.
- (vii) Joshua has played in a duo with each of the three saxophonists.
- (viii) For every pianist who has played in a duo with a saxophonist, there is another pianist who hasn't played in a duo with anyone except Pat.

- (ix) Provided that he has played in a trio with Pat and Joshua, Brad must have played in a trio with some musicians.
- (x) The three possible duets that can be formed from Brad, Joshua and Pat have all occurred, though there are some musicians with whom none of them have played. [50]

(b) Show that each of the following claims is true.

- (i) $\forall x \forall y (Rxy \rightarrow Ryx), \forall x \forall y \forall z ((Rxy \wedge Ryz) \rightarrow Rxz) \models \forall x Rxx$
- (ii) $\exists x (Fx \wedge \forall y (Fy \rightarrow x = y)), \exists x \exists y (Gx \wedge Gy \wedge \neg x = y) \models \exists x \exists y \exists z (\neg x = y \wedge \neg y = z \wedge \neg x = z \wedge \forall w ((Fw \vee Gw) \rightarrow (w = x \vee w = y \vee w = z)))$
- (iii) $\forall x \exists y (Fx \rightarrow (Fy \wedge \neg x = y)) \models \exists x \exists y (Fx \wedge Fy \wedge \neg x = y)$
- (iv) $\exists x \neg \exists y Lxy, \forall x \exists y Lxy, \forall x \forall y \forall z ((Lxy \wedge Lyz) \rightarrow Lxz) \models \exists x \forall y \forall z ((Lxy \wedge Lxz) \rightarrow y = z)$ [40]

(c) 'Strictly, the universal quantifier could be eliminated from the language of FOL: whenever we find " $\forall x Fx$ ", we could always replace it with " $Fa \wedge Fb \wedge Fc \wedge \dots$ " for every name in the language. The only reason we have the universal quantifier is therefore convenience, since writing down the corresponding conjunction would take too long.' Is this argument convincing? [10]

3. (a) Write down the axiom of extensionality. Define the terms 'union', 'intersection', 'power set' and 'subset'. [5]

(b) Let us say that a set X is *transitive* if: $\forall Y \forall Z ((Y \in X \wedge Z \in Y) \rightarrow Z \in X)$. Write down a transitive set some of whose elements are non-empty sets. Is the intersection of any two transitive sets itself always transitive? If not, give an example. [15]

(c) Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $C = \{4, 5, 6\}$. Write out the members of:

- (i) $A \cup B$
- (ii) $A \cap (B \cup C)$
- (iii) $\{x | x \subseteq B \cap C\}$
- (iv) $\{x | B \subseteq x \subseteq A \cup C\}$ [20]

(d) Let us say that a relation R is *Euclidean* if $\forall x \forall y \forall z ((Rxy \wedge Rxz) \rightarrow Ryz)$. Being careful to specify the domain in each case, give examples of the following. In each case, briefly justify your answer:

- (i) A relation that is Euclidean and transitive but not symmetric
- (ii) A relation that is Euclidean and reflexive but not transitive
- (iii) A relation that is the ancestral of a Euclidean relation [30]

(e) Let us say that a relation is *functional* if $\forall x \forall y \forall z ((Rxy \wedge Rxz) \rightarrow y = z)$. Over the domain of all people (living or dead), say whether each of the following

relations is reflexive, whether it is symmetric, whether it is transitive and whether it is functional. In each case that the relation fails to have one of these properties, briefly explain your answer.

- (i) x is the mother of y
- (ii) x is taller than y or y is taller than x
- (iii) x is Frege iff y is Wittgenstein [30]

4. (a) Write down the axioms of the probability calculus. Define conditional probability. State any form of Bayes's Theorem. [15]

(b) I have two coins, A and B. Coin A is fair. Coin B lands heads on 80% of tosses. I toss a coin and it lands heads. What is the probability that it is coin A? [20]

(c) I draw two cards from a standard pack of cards (52 cards, 13 cards in each of 4 suits) without replacement. What is the probability that they have the same number i.e. both are twos, both threes,..., both queens, both kings or both aces? [20]

(d) An urn contains one ball, which is either red or black with equal probability. Without looking, I place a second ball in the urn which I know to be red. I shake the urn and draw a ball at random. I see that it is red. What is the probability that the original ball was red? [20]

(e) I have two urns, A and B, and each contains two balls. Each urn independently has a 50% probability of containing two black balls and a 50% probability of containing one black ball and one red ball. I draw one ball from each urn and learn only that they are the same colour. What is the probability that at least one of the urns originally contained two black balls? [25]

Section B

5. How does Kripke's semantics of names allow for necessary truths that are not knowable *a priori*?
6. According to Russell, why is the sentence 'The present king of France is bald' false?
7. Distinguish conversational from conventional implicature. Can either be used to solve the paradoxes of material implication?
8. EITHER (a) Can *intentional* theories of meaning account for our understanding of unuttered sentences?
OR (b) Can *conventional* theories of meaning account for our understanding of unuttered sentences?

END OF PAPER