PHILOSOPHY TRIPOS   Part II

THURSDAY 31 May 2018   09.00 – 12.00

Paper 7

MATHEMATICAL LOGIC

Answer **three** *questions only.*

Write the number of the question at the beginning of each answer. If you are answering the either/or question, indicate the letter as well.

STATIONERY REQUIREMENTS
20 Page Answer Book x 1
Rough Work Pad

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1. How does the expressive power of first-order logic with identity differ from that of second-order logic? Why does it matter?

2. Outline the following three theories: first-order Peano arithmetic, first-order complete arithmetic, second-order Peano arithmetic. Can we say that any of them is 'best'?

3. Outline a proof of the completeness of some deductive system for first-order logic without identity. Explain how the compactness of the logic follows.

4. In a set theory without ur-elements, outline (i) a representation of the natural numbers, and (ii) an arithmetization of the real numbers.

5. Attempt all parts of this question:
   (a) Show that **R** (the set of real numbers) is uncountable.
   (b) Show that there is an injection from the Cartesian product **R**×**R** to **R**.
   (c) Show that there is a set whose cardinality is greater than **R**.

6. Does the iterative conception of set solve the set-theoretic paradoxes?

7. 'Skolem's paradox means that it is impossible to convince a determined sceptic that there really are uncountably many sets'. Discuss.

8. Could we recognize a function as computable without showing it to be recursive?

9. What kind of proof of the consistency of first-order Peano arithmetic is important for Hilbert's programme?

10. Sketch a proof that first-order Peano arithmetic does not derive its own consistency statement.

END OF PAPER