PHILOSOPHY TRIPOS   Part II

Tuesday 24 May 2016   09.00 – 12.00

Paper 7

MATHEMATICAL LOGIC

Answer **three** *questions only.*

Write the number of the question at the beginning of each answer. If you are answering the either/or question, indicate the letter as well.

STATIONERY REQUIREMENTS
20 Page Answer book x 1
Rough Work Pad

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1. Outline a proof of Löb's theorem, highlighting any assumptions you need to make about the provability predicate of Peano Arithmetic (PA). Then use Löb's theorem to derive the second incompleteness theorem for PA.

2. Let S be the set of sentences of first-order logic with identity that are not logical truths. Outline a proof that S is not effectively enumerable.

3. EITHER (a) 'The use-theoretic account of the notion of Natural Number is untenable because, by Gödel's theorem, at any given time there are truths that are not derivable from the principles of arithmetic we accept at that time.' Discuss.

   OR (b) Does Gödel's theorem show that our minds are not machines?

4. Outline a proof of the completeness of some deductive system for first-order logic with identity.

5. Say whether each of the following statements is true or false. Explain your answers, using examples where appropriate.

   (i)   Complete first-order arithmetic is categorical.

   (ii)   Some but not all axiomatisable theories are decidable.

   (iii)   If a first-order theory has arbitrarily large finite models, then it has an infinite model.

   (iv)   There is a second order formula that is true in all and only finite models.

   (v)   Full second-order logic is not weakly axiomatisable.

6. Are second-order logic and plural logic just 'set theory in sheep's clothing'?

7. Is Church's thesis an empirical claim?

8. Say whether each of the following statements is provable, refutable or undecided by standard axiomatic set theory. Explain your answers, using examples where appropriate.

   (i)   There is no universal set.

   (ii)   There is a greatest cardinal

   (iii)   Cardinal addition is commutative.

   (iv)   No ordinal is a member of itself.

(v)   There is a set of all ordinals.

9. Is the iterative concept of set hopelessly circular?

10. Arithmetic can be formalised in set theory, but axiomatic set theory can also
    be formalised within arithmetic.   Does this imply that both are alternative
    presentations of the same concepts?

END OF PAPER